MATH FOR NURSING

RATIOS AND PROPORTIONS

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Section 1: Introduction

In any field, size comparisons are often necessary. A useful method of comparing quantities is by a **ratio**, which is a concise way to express the relative sizes of two measures. There are many examples of ratios that are encountered. For example, one home remedy cleaning mixture calls for 2 cups of water to 1/4 cup of vinegar, another cleaning solution calls for 3 parts water to 1 part hydrogen peroxide. School districts post their teacher-to-student ratios and hospitals post their patient-to-nurse staffing ratios on their websites as indicators of quality.

When a ratio compares the measures of two different things it is called a **rate**. In a rate the units are different. When you have a ratio of money to the measure of a quantity you have a rate. For example, 12 ounces for $2.35 is a rate. Another common example is 55 miles per hour or 32 miles per gallon.

Conversion factors are also ratios. For example, you know that there are 12 inches in one foot. The ratio is 12 inches to 1 foot, which can also be written as 12:1 or as $\frac{12\ inches}{1\ foot}$.

A **proportion** is a statement of equality between two ratios. The term **proportional** is used to indicate that one quantity is a multiple of another. For example, when converting feet to inches we use $inches = 12 \times (number\ of\ feet)$. Another way of expressing this is to say that the number of inches is **proportional** to the number of feet. When quantities are proportional, as one variable doubles, triples, halves, etc., then so does the other.

Section 2: Using Ratios to Compare Quantities

Let's take a closer look at ratios. There are three common ways ratios are written:

$$\frac{a}{b},\ a\ to\ b,\ a:b$$

Ratios relate two quantities in different ways; part to part, part to total, and total to part. For example, in a home remedy cleaning mixture, there are 3 parts water to 1 part hydrogen peroxide. We can determine several different ratios related to this home remedy.

a) water to hydrogen peroxide 3:1
b) hydrogen peroxide to water 1:3
c) water to total mixture 3:4
d) hydrogen peroxide to total mixture 1:4
e) total mixture to water 4:3
f) total mixture to hydrogen peroxide 4:1

Example 1 : Another home remedy cleaning mixture calls for 2 cups of water to 1/4 cup of vinegar.

a) What is the ratio of vinegar to water?

b) What is the ratio of vinegar to total mixture, written as a fraction in lowest terms?

Solution: a) 1/4: 2

b) $\dfrac{\frac{1}{4}}{2\frac{1}{4}}$ This is not in lowest terms since it is a complex fraction.

To simplify, divide the numerator by the denominator.

$$\frac{\frac{1}{4}}{2\frac{1}{4}} = \frac{1}{4} \div 2\frac{1}{4} = \frac{1}{4} \div \frac{9}{4} = \frac{1}{4} \bullet \frac{4}{9} = \frac{1}{9}.$$

Therefore, for every cup of vinegar, there are 9 cups of mixture.

♦

Example 2 : In 2013 there are 2,741 females and 2046 males at Keene State College. What is the ratio of females to males?

Solution: $\dfrac{2741\,\text{females}}{2046\,\text{males}}$

This ratio could also be written as $\dfrac{1.34\,\text{females}}{1\,\text{male}}$ or just 1.34, which tells us that there are approximately one-and-one-third times as many females at Keene State as males.

Activity 1

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a) The ratio of smokers to non-smokers in a Health Science class is 3:19.

i) What is the ratio of non-smokers to smokers?

ii) Write the ratio of non-smokers to the total number of students in the class as a fraction.

b) Out of 150 students surveyed, 105 belong to the Rec Center.

i) Write the ratio of students surveyed to the students who do not belong to the Rec Center as a fraction in lowest terms.

ii) Interpret the results from part a) by filling in the blanks:

For every _____ students surveyed, _____ of them do not use the Rec Center.

Section 3: Proportions

We mentioned in the introduction that a proportion is a statement of equality between two ratios, $\frac{a}{b} = \frac{c}{d}$ is a proportion provided $b \neq 0$ and $d \neq 0$.

Two ratios are equal when the **cross products** are equal.

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } ad = cb.$$

Let's look at some examples of this.

$$\frac{3}{4} = \frac{9}{12} \text{ since } 3(12) = 4(9).$$

$$\frac{6}{7} \neq \frac{18}{20} \text{ since } 6(20) \neq 7(18).$$

Proportions can be transformed into linear equations, and we can solve them by using methods of linear equations.

Example 3 : Solve the following proportion.

$$\frac{3}{8} = \frac{x}{48}$$ (Read "3 is to 8 as x is to 48")

Solution: $\frac{3}{8} = \frac{x}{48}$

$48\left(\frac{3}{8}\right) = 48\left(\frac{x}{48}\right)$ Multiply both sides by 48 to eliminate fractions. Remember you are really multiplying by $\frac{48}{1}$.

$\frac{48(3)}{8} = x$

$x = \boxed{18}$ ♦

You could also solve the last problem by using cross products.

$\frac{3}{8} = \frac{x}{48}$ if and only if $3(48) = 8x$

$8x = 3(48)$

$x = \frac{3(48)}{8} = \frac{144}{8} = 18$

Example 4 : Solve $\frac{t}{7} = \frac{3}{5}$

Solution:

$\frac{t}{7} = \frac{3}{5}$ if and only if $5t = 3(7)$

$5t = 21$

$t = \frac{21}{5}$ ♦

Activity 2

Solve the following for x.

a) $\frac{x}{4} = \frac{0.25}{1}$

b) $\frac{4.5}{6} = \frac{15}{x}$

Example 5 **:** On a map 1 inch represents 15 miles. Find the distance represented by 3.5 inches.

Solution: We set up the proportion

$$\frac{1\ in}{15\ mi} = \frac{3.5\ in}{x\ mi}$$ (1 in is to 15 mi as 3.5 in is to ? mi)

$1x = 15(3.5)$ (cross multiply)

$x =$ 52.5 miles ♦

Note: It is important to keep your units in the proportion because you can tell if the proportion is set up properly. Notice in our example, inches are in the numerator in both ratios. If you had inches in the numerator in one and miles in the numerator in the other you would know something was wrong.

Example 6 **:** It was advertised in the newspaper that 64 oz of milk cost $2.50. If you look at the ratio amount to cost and simplified, you would get

$$\frac{64\ \text{oz}}{2.50\ \text{dollars}} = \frac{25.6\ \text{oz}}{1\ \text{dollar}}.$$

The units tell us how to interpret the ratio, we get 25.6 oz per $1.00.

We could also look at the ratio cost to amount.

$$\frac{2.50\ \text{dollars}}{64\ \text{oz}} = \frac{0.039\ \text{dollars}}{1\ \text{oz}}.$$

Interpreting this ratio, it means that for about 4 cents, we get 1 oz of milk.

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Example 7 : The ratio of females to males in Mr. Smith's A&P class is 4:3. If there are 91 students in the class, how many are female?

Solution: To set up the proportion, we need to first add the number of males and females to determine the total number of students represented by the given ratio. Next, we need to make sure that are units are consistent. Finally, we solve the proportion for the unknown quantity.

$$\frac{4\ females}{7\ students} = \frac{x\ females}{91\ students}$$

$$7x = 4(91)$$

$$7x = 364$$

$$x = 52\ females$$

Activity 3

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a) A 17.3 oz box of bran flakes costs \$2.89 and a 12 oz box costs \$1.99. Which is the better buy?

b) If a copy machine can copy 75 pages in 3 minutes, how many minutes will it take to copy 490 pages?

c) A patient wants to lose weight and determines that he can burn 75 calories for every 15 minutes that he walks. If he walks for 100 minutes, how many calories does he burn?

d) At the Health Care clinic there is a ratio of 6 nurses for every 27 patients. If the newly opened Medical Center adopts the same ratio of nursing staff to patients, how many nurses should be on duty with 45 patients on the unit?

Exercises for Ratios and Proportions

Do all the exercises on separate paper, showing all work neatly.

1. If there are 18 females and 15 males in class, what is the ratio of females to total students?

2. What is the ratio of the height of a tree 60 ft tall and the length of its shadow, 86 ft long?

3. A cupcake recipe asks for $1\frac{3}{4}$ cups of flour and $\frac{2}{3}$ cups of sugar. What is the ratio of flour and sugar?

4. Solve the following proportions.

   a) $\frac{y}{7} = \frac{5}{6}$ b) $\frac{6}{44} = \frac{15}{x}$

5. A map has a scale of 2 inches for every 20 miles. If two cities are 18.5 inches apart on the map, what is the actual distance between them?

6. If the ratio of teachers to students in a school is 1:24. How many students attend the school with 120 teachers?

7. A child receives 5 mL of Robitussin DM every 4 hours for a cough. How many mL does the child receive over a 24 hour period?

8. In a solution of water and vinegar, the ratio of water to vinegar is 5:2.

   a) Write this ratio as a fraction.

   b) If there are 30 mL of water in the solution, how many mL of vinegar are in the solution?

   c) If there are 84 mL of solution, how many mL of water are in the solution?

9. If you can type 165 words in 3 minutes and your friend can type at the same rate, how long will it take your friend to type 660 words?

RATIOS AND PROPORTIONS

Activity 1

a) i) 19:3

ii) To find the total number of students, add the non–smokers and the smokers to get 22 then write the ratio; 19:22. Next write the ratio as a fraction: $\frac{19}{22}$.

b) i) To find the number of students who do not belong to the Rec Center, subtract the number of students who do belong from the total number surveyed. 150 – 105 = 45 surveyed do not belong.

Next, write as a fraction and simplify. $\frac{total\ surveyed}{do\ not\ belong} = \frac{150}{45} = \frac{10}{3}$.

ii) For every __10__ students surveyed, __3__ of them do not use the Rec Center.

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Activity 2

a) $\frac{x}{4} = \frac{0.25}{1}$

$1x = 4(0.25)$

$x = 1$

b) $\frac{4.5}{6} = \frac{15}{x}$

$4.5x = 90$

$x = 20$

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Activity 3

a) $\frac{\$2.89}{17.3\ oz} \approx \$0.167\ per\ ounce$ $\frac{\$1.99}{12\ oz} \approx \$0.166\ per\ ounce$

Therefore, the 12 ounce box is a better buy.

b) $\frac{75\ pages}{3\ \text{min}} = \frac{490\ pages}{x\ \text{min}}$

$75x = 3(490)$

$75x = 1470$

$x = 19.6\ \text{min}$

Therefore, it will take 19.6 minutes to copy 490 pages.

Activity 3 continued:

$$\frac{75\ calories}{15\ \text{min}} = \frac{x\ calories}{100\ \text{min}}$$

c) $15x = 75(100)$

$15x = 7500$

$x = 500\ calories$

Therefore, he will burn 500 calories in 100 minutes.

$$\frac{6\ nurses}{27\ patients} = \frac{x\ nurses}{45\ patients}$$

d) $27x = 6(45)$

$27x = 270$

$x = 10\ nurses$

Therefore, 10 nurses should be on duty for 45 patients.