# MATH FOR NURSING



# DECIMALS

Written by: Joe Witkowski and Eileen Phillips

### **Decimals**

# Section 1: Introduction

The name and the place value for the first seven columns in our number system is as follows:

	Hundred	Ten				
Millions	Thousands	Thousands	Thousands	Hundreds	Tens	Ones
Column	Column	Column	Column	Column	Column	Column
1,000,000	100,000	10,000	1,000	100	10	1

As we move from right to left, we multiply by 10 each time. The value of each column is 10 times the value of the column on its right, with the rightmost column being 1. To understand the idea behind decimal numbers, we notice that moving in the opposite direction, from left to right, we divide by 10 each time.

If we are at the ones column and go to the right the next column would have to be

$$1 \div 10 = \frac{1}{10}$$
 Tenths

The next one after that will be

$$\frac{1}{10} \div 10 = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$$
 Hundredths

After that, we have

$$\frac{1}{100} \div 10 = \frac{1}{100} \cdot \frac{1}{10} = \frac{1}{1000}$$
 Thousand**th**s

We could continue this process of dividing by 10 to move one column to the right as long as we wanted to. A **decimal point** is used to show where the ones column is. The decimal point is placed between the ones column and the tenths column.

We use the place value of decimals to write them in expanded form.

**Example 1**: Write 321.765 in expanded form.

**Solution**: 
$$321.765 = (3 \times 100) + (2 \times 10) + (1 \times 1) + (7 \times \frac{1}{10}) + (6 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$$

or 
$$321.765 = (3 \times 100) + (2 \times 10) + (1 \times 1) + \frac{7}{10} + \frac{6}{100} + \frac{5}{1000}$$

**Example 2**: Write each number in words.

- a) 0.3
- b) 0.03
- c) 0.003

**Solution**: a) 0.3 is "three tenths."

- b) 0.03 is "three hundredths."
- c) 0.003 is "three thousandths."

When a decimal contains digits to the left of the decimal point, the word "and" is used to indicate where the decimal point is when writing the number in words.

**Example 3**: Write the following in words.

- a) 4.2
- b) 4.02
- c) 4.002

**Solution:** a) 4.2 is "four and two tenths."

- b) 4.02 is "four and two hundredths."
- c) 4.002 is "four and two thousandths."

**Example 4**: Write 5.23 in words.

**Solution:** The number 5.23 is read "five and twenty-three hundredths."

ones tenths hundredths

The decimal part is read as "twenty-three hundredths" because

2 tenths + 3 hundredths = 
$$\frac{2}{10}$$
 +  $\frac{3}{100}$  =  $\frac{20}{100}$  +  $\frac{3}{100}$  =  $\frac{23}{100}$ 

# **Activity 1**

A. Write out the name of each of the numbers in words.

- 1. 1.73
- 2. 0.005

B. Write each number in decimal notation.

- 1. Eight and ninety-three thousandths
- 2. Sixteen and one hundred nine ten-thousandths

**Example 5**: Write each number as a fraction or a mixed number. Do not simplify.

- a) 0.003
- b) 15.6547

**Solution**: a) Since 0.003 is 3 thousandths, we write

$$0.003 = \frac{3}{1000}$$

b) 
$$15.6547 = 15 \frac{6547}{10000}$$

# **Activity 2**

Write each number as a fraction or a mixed number.

- 1. 9.019
- \_\_\_\_\_
- 2. 78.143

\_\_\_\_\_

3. 0.45

\_\_\_\_

Example 6:

Write the following decimal numbers in order from smallest to largest.

6.02, 6.24, 6.0024, 6.204, 6.04

**Solution:** 

1. Line up the decimal points.

6.02 6.24 6.0024

6.0022 6.204 6.04

2. Fill in with zeros so that all decimals have the same number of places.

6.0200

6.2400

6.0024

6.2040

6.0400

- 3. Since all the ones column contain a 6, we ignore that column and treat the decimal portions as if they were whole numbers. Then compare those numbers, putting them in order from smallest to largest: 24, 200, 400, 2040, and 2400.
- 4. Write the decimals in order:

6.0024, 6.02, 6.04, 6.204, 6.24

## **Activity 3**

Write the following decimal numbers in order from smallest to largest.

a) 0.05 0.04 0.045 0.004 0.0405

b) 0.017 0.17 0.0071 0.07 0.01

# **Section 2: Adding and Subtracting Decimals**

The algorithm for adding decimals is a three step process:

- 1. Line up decimal points. (Add zeros if necessary.)
- 2. Add, ignoring decimal points.
- 3. Insert a decimal point in the sum directly below those in the addends.

You can see from this example, that adding decimals is almost as easy as adding whole numbers. Let's try to understand **why** the algorithm works. The small numbers above the equal sign correspond to the steps in the algorithm.

$$2.345 + 45.5 = \frac{2345}{1000} + \frac{455}{10}$$

$$\frac{1}{2} = \frac{2345}{1000} + \frac{45500}{1000}$$
raising a fraction to higher terms
$$\left(\frac{a}{b} = \frac{a \cdot 100}{b \cdot 100}\right)$$

$$= \frac{2345 + 45500}{1000}$$
adding fractions with a common denominator
$$\frac{2}{5} = \frac{47845}{1000}$$
adding whole numbers
$$\frac{3}{5} = 47.845$$
dividing by 1000

**Example 8**: Add by first changing to fractions: 25.43 + 2.897

**Solution**: 
$$25.43 + 2.897 = 25\frac{43}{100} + 2\frac{897}{1000}$$

$$= \frac{2543}{100} + \frac{2897}{1000} = \frac{25430}{1000} + \frac{2897}{1000}$$

$$= \frac{25430 + 2897}{1000}$$

$$= \frac{28327}{1000}$$

$$= 28.327$$

Now let's add the numbers in the last example without changing them to fractions.

Note: The decimal point in the answer is directly below the decimal points in the problem.

**Example 10**: Add 
$$5 + 1.0034 + 18.75 + 6.4$$

The algorithm for **subtracting** decimals is essentially the same as the addition algorithm:

- 1. Line up decimal points. (Add zeros if necessary.)
- 2. Subtract, ignoring decimal points.
- 3. Insert a decimal point in the difference directly below those in the minuend and subtrahend.

The explanation of **why** this algorithm works is essentially the same as the explanation for addition.

**Solution**: As before we will first line numbers up vertically, with the decimal points lined up, and then subtract as usual. Remember to place the decimal point in the answer directly below the decimal point in the problem.

**Example 12**: Simplify: 
$$45.6 - 4.3 + 6.8 - 5$$

**Solution:** Working from left to right:

1. 
$$45.6$$
 2.  $41.3$  3.  $48.1$   $\frac{-4.3}{41.3}$   $\frac{+6.8}{48.1}$   $\frac{-5.0}{43.1}$ 

Therefore, 
$$45.6 - 4.3 + 6.8 - 5 = 43.1$$

**Activity 4** 

**Example 13**: Add 4.75 + (-7.23)

**Solution:** Recall that to add two numbers with different signs, we have to subtract the smaller absolute value from the larger. The sign of the answer is the same as the sign of the number with the larger absolute value.

$$|4.75| = 4.75$$
 and  $|-7.23| = 7.23$ 

Now subtract the smaller absolute value from the larger absolute value.

Since 7.23 is larger than 4.75, we keep the sign of 7.23.

Therefore, 4.75 + (-7.23) = -2.48

**Example 14** : Subtract -5 - 2.34

**Solution**: -5 - 2.34 = -5 + (-2.34) Since subtraction can be thought of as addition of the opposite.

To add two numbers of the same sign, we add their absolute values and keep the sign of the original two numbers.

$$-5 - 2.34 = -7.34$$

$$5.00$$

$$+ 2.34$$

$$-2.34$$

$$-3.34$$
Add the absolute values of the two numbers.

The answer has the same sign as the original two numbers.

# **Activity 5**

1. Add -3.73 + 6.67

2. Subtract -9.67 - 14

# **Section 3: Multiplying Decimals**

Like the algorithm for adding and subtracting decimals, the multiplication algorithm also involves three steps.

- 1. Multiply, ignoring decimal points.
- 2. Add up the number of decimal places in the factors.
- 3. Insert a decimal point in the product so that

$$\begin{bmatrix} \text{Number of } \\ \text{decimal places} \\ \text{in product} \end{bmatrix} = \begin{bmatrix} \text{Number of } \\ \text{decimal places} \\ \text{in first factor} \end{bmatrix} + \begin{bmatrix} \text{Number of } \\ \text{decimal places} \\ \text{in second factor} \end{bmatrix}$$

**Solution:** 

The decimal point is placed so there are 3 + 2 = 5 decimal places.

Let's take a closer look at the multiplication algorithm to try and understand why it works. The multiplication algorithm relies on properties of fractions. As before, the small numbers above the equal sign refer to the steps in the algorithm.

$$1.751 \times 2.34 = \frac{1751}{1000} \times \frac{234}{100}$$

$$= \frac{1751 \times 234}{1000 \times 100}$$
 rule for multiplying fractions
$$\frac{1.2}{100000} = \frac{409734}{100000}$$
 multiplying whole numbers
$$\frac{3}{100000} = 4.09734$$
 dividing by 100000

The multiplication algorithm is a shortcut so we do not have to first change the decimal to a fraction.

Example 16 :

How many digits will be to the right of the decimal point in the following product?

**Solution**:

There are three digits to the right of the decimal point in 4.005, and two digits to the right of the decimal point in 0.65. Therefore, there will be 3 + 2 = 5 digits to the right of the decimal point in the product.

Example 17 :

Simplify: 
$$(0.15)^2$$

**Solution:** 

Recall that the exponent of 2 tells us to multiply the base, 0.15, by itself.

Since we had 4 decimal places in our factors, we needed to add a zero in front of the 3 to have 4 decimal places in our answer.

Example 18 :

Multiply 4.65 x 2.67

**Solution**:

# **Activity 6**

Find each of the following products.

4. 
$$(0.23)^2$$

A common error that happens when multiplying decimals is to place the decimal point in the wrong place. You should always estimate to make sure the decimal was placed in the proper place. For example, suppose you were multiplying 45.6 by 2.3 and came up with the following:

If you estimate the answer, it is then easy to place the decimal. In this example we have about 45 times 2, so the decimal point would only make sense between the 4 and 8 giving an answer of 104.88.

## **Activity 7**

Use estimation to place the decimal point in the product.

a) 
$$4.36 \times .4 = 1744$$
 b)  $52 \times .19 = 988$  c)  $52 \times 1.9 = 988$ 

c) 
$$52 \times 1.9 = 988$$

# Section 4: Dividing Decimals

Division of decimals closely follows division of whole numbers. Suppose you had to find  $123.45 \div 2.5$ . The traditional division algorithm for decimals is as follows:

1. Set up the calculation in the usual whole number format.

2 Move the decimal point in the divisor just enough places to the right so that the new divisor is a whole number, and move the decimal point in the dividend the same number of places to the right:

Divide as usual, and insert the decimal point in the quotient directly above the 3. decimal point in the (new) dividend:

$$\begin{array}{r} 49.38 \\ 25 \overline{\smash)1234.50} \\ \underline{100} \\ 234 \\ \underline{225} \\ 95 \\ \underline{75} \\ 200 \\ 200 \end{array}$$

0 <-- If a remainder of zero was not found, more zeros could be added to the dividend until a remainder of zero was found or instructions were given to round to a specific place value.

Let's try to see why this algorithm works. In fraction form, the problem above is equivalent to  $\frac{123.45}{2.5}$ . If we want to write the divisor as a whole number, we can multiply the numerator and the denominator of this fraction by 10:

$$\frac{123.45 \times 10}{2.5 \times 10} = \frac{1234.5}{25}$$

Since this fraction is equivalent to the original fraction, our original division problem is equivalent to 25 1234.5. This justifies step 2 in the division algorithm.

To see why step 3 works, we note that the answer has to check. In other words,

quotient x divisor = dividend.

By what we know about multiplication of decimals, this equation implies that

That is,

In other words, to find the number of decimal places in the quotient, one must decrease the number of decimal places in the dividend by the number in the divisor. This is what is accomplished by sliding decimal points over and up.

**Example 19** : Divide 1 by 62.5

Step 3 625 
$$\begin{vmatrix} 0.016 \\ 10.000 \end{vmatrix}$$
 Divide as usual, and insert the decimal point in the quotient directly above the decimal point in the (new) dividend.

# **Activity 8**

Divide the following:

1. 
$$17.759 \div 3.01$$

2. 
$$1.53 \div 7.5$$

# Section 5: Changing Fractions to Decimals

A fraction can be thought of as a numerator divided by a denominator. For example, the fraction  $\frac{3}{5}$  can be thought of as 3 divided into 5 equal parts, or simply as 3 divided by 5. So to change a fraction to a decimal you just have to carry out the division.

**Example 20**: Change 
$$\frac{3}{5}$$
 to a decimal.

**Example 21**: Change 
$$\frac{7}{8}$$
 to a decimal.

**Solution:** We have to divide 7 by 8. It is always a good idea to have an estimate of the answer before we begin. Since 
$$\frac{7}{8}$$
 is greater than  $\frac{1}{2}$  and less than 1, the answer has to be between .5 and 1.

$$\begin{array}{r}
 .875 \\
 7.000 \\
 \underline{64} \\
 60 \\
 \underline{56} \\
 40 \\
 \underline{40} \\
 0
\end{array}$$

Therefore, 
$$\frac{7}{8} = .875$$

Many times when changing a fraction to a decimal, the decimal does not terminate like the last two examples. For example,  $\frac{1}{3} = .333333...$ , where the three dots mean that the decimal never terminates.

Another way to write this is  $\frac{1}{3} = .\overline{3}$  where the line goes above the part that repeats.

**Example 22**: 
$$\frac{1}{7} = .\overline{142857}$$

This means that the .142857 repeats indefinitely, in other words  $\frac{1}{7} = .142857142857142857142857...$ 

# **Activity 9**

Change the following fractions to decimals.

- a)  $\frac{7}{20}$
- b)  $\frac{1}{40}$
- c)  $\frac{5}{6}$
- d)  $\frac{3}{11}$

**Example 23**: Add: 
$$0.12 + \frac{3}{8}$$

FRACTIONS: 
$$0.12 = \frac{12}{100}$$

$$\frac{12}{100} + \frac{3}{8} = \frac{24}{200} + \frac{75}{200} = \frac{99}{200}$$

DECIMALS: First change 
$$\frac{3}{8}$$
 to a decimal.

$$\begin{array}{r}
375 \\
8)3.000 \\
\underline{24} \\
60 \\
\underline{56} \\
40 \\
\underline{40} \\
0
\end{array}$$

$$0.12 + \frac{3}{8} = 0.12 + 0.375 = \mathbf{0.495}$$

# Activity 10

Show that 
$$\frac{99}{200} = 0.495$$

# **Section 6: Rounding Decimals**

The rule for rounding decimal numbers is similar to the rule for rounding whole numbers.

- 1. If the digit in the column to the right of the one you are rounding to is 5 or more, add 1 to the digit in the column we are rounding to; otherwise, we leave it alone.
- 2. Replace all digits to the right of the column we are rounding to with zeros if they are to the left of the decimal point; otherwise, we delete them.

**Example 24**: Round 467.999 to the nearest ten.

**Solution**: The number next to the tens column is 7, which is 5 or more, so we add 1 to 6. We change all digits to the right to 0, and drop all digits to the right of the decimal point.

467.999 is rounded to **470** 

**Example 25**: Round 1.006349 to the nearest ten thousandth.

**Solution:** Since the number to the right of the ten thousandths column is 4, which is less than 5, the 3 is left alone. All digits to the right of 3 are deleted.

1.006349 is rounded to **1.0063** 

# **Activity 11**

1. Complete the following table.

	Rounded to the Nearest					
Number	Whole number	Tenth	Hundredth			
4.099						
4.936						
0.074999						
0.545						

- 2. Round 9.05496 to the nearest ten thousandth.
- 3. Round 10,479.057 to the nearest thousand.

Example 26:

Divide and round the answer to the nearest hundredth.

$$23.002 \div 3.01$$

**Solution:** 

Since we are asked to round to the hundredth place, we must carry out our division to the thousandth place (3 decimal places). We will use the thousandths place digit to round the hundredths place digit.

$$\begin{array}{r}
 7.641 \\
 3.01 \overline{\smash{\big)}\ 23.002000} \\
 \underline{2107} \\
 1932 \\
 \underline{1806} \\
 1260 \\
 \underline{1204} \\
 560 \\
 \underline{301} \\
 259
\end{array}$$

Since the number to the right of the 4 in the quotient is less than 5, the 4 is left alone. Therefore,

$$23.002 \div 3.01$$
 is rounded to **7.64**

# Activity 12

Divide and round the answer to the nearest hundredth.

$$105.208 \div 7.9$$

# **Exercises for Decimals**

# Do all the exercises on separate paper, showing all work neatly.

4	XX7 11		C 1	1	•	1
	Write the	name	ot each	number	1n	words
1.	WILL LIIC	manne	or cacin	Hullioti	111	words.

- a) 0.103
- b) 5.02
- c) 24.6

#### 2. Write each number as a fraction or a mixed number. Do not simplify your answers.

- a) 2.96
- b) 65.101
- c) 0.00062

- a) 314.67
- b) 65.12
- c) 0.0021
- d) 8.54321

- a) Six and two tenths
- b) two hundred and 8 hundredths
- c) Five thousand and five thousandths
- d) Twelve thousand and four hundred thirty-five thousandths

a)

b)

.03

5.7

.02

5.07

- .025 .0099

5.27

5.027 5.207

- a) 0.35
- b) 0.125
- c) 0.0605

0.209

- d) 0.0500
- e) 0.1742

4.56 + 2.09a.

0.04 + 0.83 + 0.72b.

- c. 5.0004 + 2.97 + 0.008
- d. 0.81 + (-5) + 4.95
- e. 65.0197 + 6.78 + 0.0009
- f. 987.658 + 341.396

g. 76.56 - 49.82

h. 5.94 - 4.84

i. -5 - .943

- j. 15.837 + 19.02 + 7 + .49
- k. 30.7 + 8.042 6.3 + 2.19
- 1.  $1.43 + \frac{3}{4}$

m.  $\frac{2}{5} + 0.63 - \frac{1}{8}$ 

n.  $(0.35)^2$ 

o.  $(1.4)^2$ 

q. 411.4 ÷ 44

r. 2.3 x 4.52

s. 21.978 ÷ 3.3

t.  $2.40 \div 0.75$ 

- u. 4.005 x 0.97
- 8. a. Round 2,456.8706 to the nearest thousandth.
  - b. Round 0.6235 to the nearest hundredth.
  - c. Round 15.8479 to the nearest thousandth.
  - d. Round 109.543 to the nearest ones.
- 9. Write each fraction as a decimal.
  - a.  $\frac{4}{5}$
- b.  $\frac{1}{6}$
- c.  $\frac{9}{80}$
- d.  $\frac{3}{7}$

#### Activity 1:

- A. 1. one and seventy-three hundredths B. 1. 8.093

2. five thousandths

2. 16.0109

#### Activity 2:

1. 
$$9\frac{19}{1000}$$

2. 
$$78\frac{143}{1000}$$

$$9\frac{19}{1000}$$
 2.  $78\frac{143}{1000}$  3.  $\frac{45}{100} = \frac{9}{20}$ 

#### Activity 3:

1. 
$$0.004 < 0.04 < 0.0405 < 0.045 < 0.05$$
 2.  $0.0071 < 0.01 < 0.017 < 0.07 < 0.17$ 

$$2. \quad 0.0071 < 0.01 < 0.017 < 0.07 < 0.17$$

#### Activity 4:

# Activity 5:

#### Activity 6:

Find each of the following products.

2. 
$$2.604 \times 14.03$$
  $2.6004$   $\times 14.03$   $7812$   $1041600$   $2604000$   $36.53412$ 

#### Activity 6 continued:

$$\begin{array}{ccc} 4. & (0.23)^2 & 0.23 \\ & \times 0.23 \\ & 69 \\ & \underline{460} \\ & 0.0529 \end{array}$$

\*

Activity 7:

a) 
$$4.36 \times 0.4 = 1.744$$
 b)  $52 \times 0.19 = 9.88$   
 $4 \times 0.4 = 1.6$   $50 \times 0.2 = 10$ 

c) 
$$52 \times 1.9 = 98.8$$
  
 $50 \times 2 = 100$ 

Activity 8:

1. 
$$301)1775.9$$

$$1505$$

$$2709$$

$$2709$$

$$0$$

Therefore,  $17.759 \div 3.01 = 5.9$ 

Therefore,  $1.53 \div 7.5 = 0.204$ 

Activity 9:

a) 
$$\frac{0.35}{20)7.00}$$

$$\frac{60}{100}$$

$$\frac{100}{0}$$

b) 
$$\frac{0.025}{40)1.000}$$
  $\frac{80}{200}$   $\frac{200}{0}$ 

#### Activity 9 continued:

$$\begin{array}{r}
 0.83 \\
6)5.00 \\
 \hline
 20 \\
 \underline{18} \\
 2
\end{array}$$

$$\frac{0.2727}{11)3.0000}$$

$$\frac{22}{80}$$

$$\frac{77}{30}$$

$$\frac{22}{80}$$

$$\frac{77}{30}$$

$$\frac{22}{80}$$

$$\frac{77}{3}$$

#### Activity 10:

$$\frac{0.495}{200)99.000}$$
Show that  $\frac{99}{200} = 0.495$ 

$$\frac{800}{1900}$$

$$\frac{1800}{1000}$$

$$\frac{1000}{0}$$

#### Activity 11:

1.

#### Rounded to the nearest

Number	Whole number	Tenth	Hundredth			
4.099	4	4.1	4.10			
4.936	5	4.9	4.94			
0.074999	0	0.1	0.07			
0.545	1	0.5	0.55			

2. Round 9.05496 to the nearest ten-thousandth 9.0550

3. Round 10,479.057 to the nearest thousand <u>10,000</u>

#### Activity 12:

Divide:  $105.208 \div 7.9$  round the answer to the nearest hundredth.

$$\begin{array}{r}
 13.317 \\
 79 \overline{\smash{\big)}\ 1052.080} \\
 \underline{79} \\
 262 \\
 \underline{237} \\
 250 \\
 \underline{237} \\
 138 \\
 \underline{79} \\
 590 \\
 \underline{553} \\
 37
\end{array}$$

Since the number to the right of 1 is 5 or more, the 1 becomes a 2. Therefore,  $105.208 \div 7.9 = 13.32$