Math 171 Proficiency Packet on Percents

Section 1: Introduction

Percentages are just another way of expressing fractions. **Percent** means "per hundred" or "divided by one hundred." For example, 25% is the same as 25 one-hundredths. In other words,

$$25\% = \frac{25}{100}$$

or written as a decimal

$$25\% = 0.25.$$

The following examples illustrate the meaning of percents.

Example 1: Write the following as equivalent fractions without the % symbol.

a) $6\% = \frac{6}{100}$

b) $1.9\% = \frac{1.9}{100}$

c) $110\% = \frac{110}{100}$

d) $45\frac{1}{3}\% = \frac{45\frac{1}{3}}{100}$

Example 2: Write the following using the % symbol.

a) $\frac{15}{100} = 15\%$

b) $\frac{9.25}{100} = 9.25\%$

You can see that percents are just fractions (or ratios) with denominator 100.

Now You Try (Section 1)

1) Write the following as an equivalent fraction without the % symbol.

a) 5% b) 7.6% c) 270%

2) Write the following using the % symbol.

a) $\frac{16}{100}$ b) $\frac{9\frac{1}{8}}{100}$

(Answers to **Now You Try** (Section 1) *are found on page 19.)*

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Section 2: Percents Using Decimal Squares

Using decimal squares, it is easy to relate percents to fractions and decimals. The following grid is a Decimal Square; it is divided into 100 equal parts, and 30 parts are shaded. Since 30 out of 100 are shaded, we can say that 30% is shaded. The shaded portion also represents the decimal .30 and the fraction $\frac{30}{100}$ or $\frac{3}{10}$.

x	x	x							
x	x	x							
x	x	x							
x	x	x							
x	x	x							
x	x	x							
x	x	x							
x	x	x							
x	x	x							
x	x	x							

Example 1: Use the Decimal Square to shade $8\frac{1}{2}\%$. Then write the decimal and fraction equivalent of $8\frac{1}{2}\%$.

Solution: $8\frac{1}{2}\%$ means $8\frac{1}{2}$ parts out of 100 equal parts, so we have to shade $8\frac{1}{2}$ of the squares.

x									
x									
x									
x									
x									
x									
x									
x									

To write $8\frac{1}{2}\%$ as a fraction and a decimal:

1) Change $8\frac{1}{2}\%$ to 8.5%. *(Note this is still a % even though there is a decimal involved.)*

2) Write 8.5% as a fraction, $\frac{8.5}{100}$.

3) Eliminate the decimal within the fraction by multiplying numerator and denominator by 10, giving us $\frac{8.5\times 10}{100\times 10} = \frac{85}{1000}$.

4) Now we can write $\frac{85}{1000}$ in lowest terms, by dividing numerator and denominator by 5, giving us $\frac{85\div 5}{1000\div 5} = \frac{17}{200}$.

5) We can write the decimal equivalent of $8\frac{1}{2}\%$ by looking at the fraction $\frac{85}{1000}$. We read this as "eighty-five thousandths" or 0.085.

Therefore, $\boxed{8\frac{1}{2}\% = 0.085 = \frac{17}{200}}$.

We can also use the Decimal Square to find the decimal and fraction equivalents visually.

The square that is half shaded is shown below, divided into tenths.

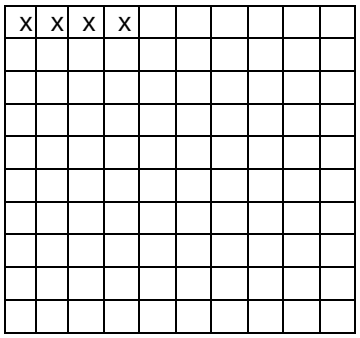
x	x	x	x	x

If all the squares in the Decimal Square were divided in the same way, we would have 1000 equal parts and 85 would be shaded. Therefore, $8\frac{1}{2}\%$ is equivalent to $\frac{85}{1000}$ or 0.085.

Now You Try (Section 2)

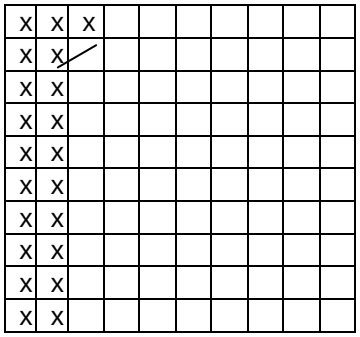
1) For each of the following Decimal Squares, determine how much is shaded. Write your answer as a percent, decimal, and a fraction.

a)



i) Percent _____ ii) Fraction _____ iii) Decimal _____

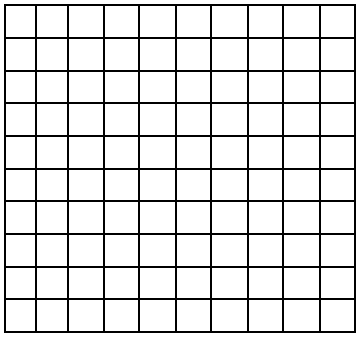
b)



i) Percent _____ ii) Fraction _____ iii) Decimal _____

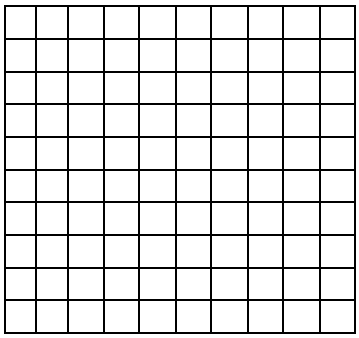
2) Shade the indicated percent. Write the percent as a decimal and as a fraction.

a) i) $15\frac{1}{2}\%$



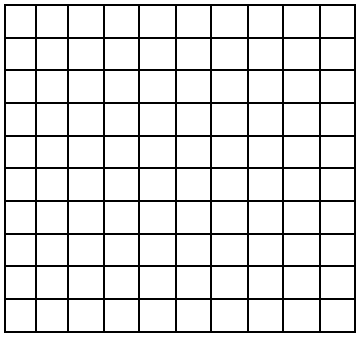
ii) Fraction _____ iii) Decimal _____

b) i) $6\frac{1}{4}\%$



ii) Fraction _____ iii) Decimal _____

3) If you were going to shade 1% in the following Decimal Square, you would shade 1 square. Shade 110% in the Decimal Square.



(Answers to **Now You Try** (Section 2) *are found on page 19.)*

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Section 3: Changing Percents to Decimals

We use the meaning of percent to change a percent to a decimal number.

Example 1: Change 37% to a decimal.

Solution: 37% means 37 "per hundred" or 37 over 100.

$37\% = \frac{37}{100}$ *We used the meaning of % to convert to a fraction with denominator 100.*

$= 0.37$ *We read* $\frac{37}{100}$ *as "37 hundredths" which in decimal form is 0.37.*

Example 2: Change 8.75% to a decimal.

Solution: 8.75% means 8.75 "per hundred" or 8.75 over 100.

$8.75\% = \frac{8.75}{100}$ *We used the meaning of % to convert to a fraction with denominator 100.*

$= \frac{875}{10000}$ *Multiplied numerator and denominator by 100 to eliminate the decimal in the numerator*

$= 0.0875$ *We read* $\frac{875}{10000}$ *as "eight hundred seventy-five ten-thousandths", which in decimal form is 0.0875.*

From the last two examples, you can see that the % symbol has been dropped and the decimal point has been moved two places to the **left**. Since % means "per hundred", you will always move the decimal two places to the left when changing a percent to a decimal.

Shortcut: **To change a percent to a decimal, drop the % symbol and move the decimal point two places to the left.**

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Now You Try (Section 3)

Change the following percents to decimals, using the shortcut given above.

1) 98%

2) 1%

3) 10.07%

4) 0.9%

5) 150%

(Answers to **Now You Try** (Section 3) *are found on page 19.)*

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Section 4: Changing Decimals to Percents

Instead of writing the fraction equivalent of a percent and then writing the decimal equivalent like we did with the Decimal Squares, we can use the previous shortcut to go directly from a percent to a decimal. To change percents to decimals we moved the decimal point two places to the left. Therefore, to change decimals to percents we have to do the opposite. For example 12% written as a decimal is 0.12, which means to change 12% back to a decimal we move the decimal two places to the right and insert the % symbol:

$0.12 = 12\%$

Shortcut: **To change a decimal to a percent, we move the decimal point two places to the right and insert the % symbol.**

Example 1: **Change the following decimals to percents.**

a) 0.45 **b) 0.005** **c) 5**

Solution: **For each example, we will use the shortcut given above.**

a) 0.45 = 45% b) 0.005 = 0.5% c) 5 = 5.00 = 500%

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Now You Try (Section 4)

Change the following decimals to percents.

1) .67 3) 0.8

2) 0.0007 4) 1.2

(Answers to **Now You Try** (Section 4) *are found on page 19.)*

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Section 5: Changing Percents to Fractions

Since we know that percent means "per hundred", to change a percent to a fraction we just **drop** the % symbol and write the original number over **100**.

Example 1: Change 40% to a fraction.

Solution: $40\% = \frac{40}{100}$ *Percent means "per hundred."*

$= \boxed{\frac{2}{5}}$. *Write the fraction in lowest terms.*

Example 2: Change 10.5% to a fraction.

Solution: $10.5\% = \frac{10.5}{100}$ *Percent means "per hundred."*

$= \frac{10.5 \times 10}{100 \times 10}$ *We want a whole number in the numerator, so we find an equivalent fraction by multiplying the numerator and denominator by 10.*

$= \frac{105}{1000}$

$= \boxed{\frac{21}{200}}$. *Write the fraction in lowest terms.*

Example 3: Change $8\frac{3}{4}\%$ to a fraction.

Solution:

$8\frac{3}{4}\% = \frac{8\frac{3}{4}}{100}$ *Percent means "per hundred."*

$= \frac{\frac{35}{4}}{100}$ *Change* $8\frac{3}{4}$ *to an improper fraction.*

$= \frac{35}{4} \div 100$ $\frac{a}{b}$ *means* $a \div b$.

$= \frac{35}{4} \times \frac{1}{100}$ *Dividing by 100 is the same as multiplying by* $\frac{1}{100}$.

$= \frac{35}{400}$ *Multiply numerators, then denominators.*

$= \boxed{\frac{7}{80}}$. *Write the fraction in lowest terms.*

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Now You Try (Section 5)

Change the following percents to fractions. Write the fractions in lowest terms.

1) 65%

2) 100%

3) 240%

4) 34.6%

5) $7\frac{1}{3}\%$

6) $\frac{1}{2}\%$

(Answers to **Now You Try** (Section 5) *are found on page 20.)*

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Section 6: Changing Fractions to Percents

We can change a fraction to a decimal either by ;

1) Changing the fraction to a decimal and then change the decimal to a percent or,

2) Using a proportion.

The following examples illustrate both methods.

Example 1: Change $\frac{9}{10}$ to a percent.

Solution: **Method 1:**

Let's first change $\frac{9}{10}$ to a decimal and then change the decimal to a percent.

We change $\frac{9}{10}$ to a decimal by dividing 9 by 10:

$$10\overline{)9.0}\quad 0.9$$
$$\underline{90}$$
$$0$$

We now have to change 0.9 to a percent by moving the decimal point two places to the right and inserting the % symbol.

Therefore, $\boxed{\frac{9}{10} = 0.9 = 90\%}$.

Method 2:

From the definition of percent, we know that percent is a fraction with a denominator of 100, $x\% = \frac{x}{100}$. Thus to express a fraction as a percent we must find an equivalent fraction that has a denominator of 100.

$\frac{9}{10} = \frac{x}{100}$ *Set up an equation with the given fraction and* $\frac{x}{100}$.

$10x = 9(100)$ *Cross-multiply.*

$\frac{10x}{10} = \frac{900}{10}$ *Divide both sides of the equation by 10.*

$x = 90$

Therefore, $\boxed{\frac{9}{10} = \frac{90}{100} = 90\%}$. *Substitute 90 in for x in the original equation.*

Example 2: Change $\frac{5}{8}$ to a percent.

Solution: **Method 1:** Divide 5 by 8.

$$8\overline{)5.000}\quad 0.625$$

$$\begin{array}{r} 48 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

Therefore, $\boxed{\frac{5}{8} = 0.625 = 62.5\%}$.

Method 2: Set up a proportion.

$$\frac{5}{8} = \frac{x}{100}$$

$$\frac{8x}{8} = \frac{500}{8}$$

$$x = 62.5 = 62\frac{1}{2}$$

Therefore, $\boxed{\frac{5}{8} = \frac{62\frac{1}{2}}{100} = 62\frac{1}{2}\%}$.

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Now You Try (Section 6)

Change the following fractions to percents.

1) $\frac{3}{4}$ 2) $\frac{17}{40}$ 3) $\frac{5}{12}$

(Answers to **Now You Try** (Section 6) *are found on page 20.)*

Section 7: Percent Problems

The general form of a percent problem is $\boxed{\%\ of\ \ total\ is\ \ part}$ or $\boxed{\frac{\%}{100} = \frac{part}{total} = \frac{is}{of}}$.

We can solve percent problems by using 2 different methods;

Method 1: Use a proportion.

Method 2: Translate into a mathematical equation.

We will use these 2 methods to solve the following percent problems.

Example 1: 15% of 40 is what number?

Solution: Here we know the percent (15%) and the total (40) and we are looking for the part.

Method 1: $\boxed{\frac{\%}{100} = \frac{part}{total} = \frac{is}{of}}$

$\frac{15}{100} = \frac{n}{40}$ *Place 15 over 100 and drop the % symbol. Substitute 40 in for the total.*

$100n = 15(40)$ *Cross multiply.*

$\frac{100n}{100} = \frac{600}{100}$ *Divide both sides by 100.*

$n = 6$

Therefore, 15% of 40 is 6 .

Method 2: Using the general form, $\boxed{\%\ of\ \ total\ is\ \ part}$, we translate the following to math symbols:

% $\rightarrow$ change to a decimal

of $\rightarrow$ change to multiplication

is $\rightarrow$ change to an equals sign

Now that we have a mathematical equation, we solve for the unknown.

To answer the given question, 15% of 40 is what number ?

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

We translate to get: .15 · 40 = n

Then we solve for n by multiplying: 6 = n

Once again, 15% of 40 is 6 .

Example 2: 25% of what number is 50.2?

Solution: Here we know the percent (25%) and the part (50.2) and we are looking for the total.

Method 1: $\boxed{\frac{\%}{100} = \frac{part}{total} = \frac{is}{of}}$

$$\frac{25}{100} = \frac{50.2}{n}$$ *Place 25 over 100 and drop the % symbol. Substitute 50.2 for the part.*

$$25n = (50.2)(100)$$ *Cross multiply.*

$$\frac{25n}{25} = \frac{5020}{25}$$ *Divide both sides by 25.*

$$n = 200.8$$

Therefore, 25% of 200.8 is 50.2.

Method 2: $\boxed{\%\ of\ total\ is\ part}$

To answer the given question, 25% of what number is 50.2?

We translate to get: 0.25 · n = 50.2

To solve for n, we must divide both sides of the equation by 0.25.

$$\frac{0.25 \cdot n}{0.25} = \frac{50.2}{0.25}$$

$$n = 200.8$$

Once again, 25% of 200.8 is 50.2.

Example 3: 45 successful free throws is what % of 68 attempts? (Round the answer to the nearest tenth of a percent.)

Solution: Here we know the part (45 successful free throws) and the total (68 attempts) and we are looking for the percent.

Method 1: $\boxed{\frac{\%}{100} = \frac{part}{total} = \frac{is}{of}}$

$$\frac{n}{100} = \frac{45}{68}$$

$$68n = 45(100)$$

$$\frac{68n}{68} = \frac{4500}{68}$$

$$n \approx 66.2\%$$

Therefore, 45 successful free throws is approximately 66.2% of 68 attempts.

Method 2: $\boxed{\%\ of\ total\ is\ part}$

What % of 68 is 45?	*Write a question for the given problem.*
$\frac{n \cdot 68}{68} = \frac{45}{68}$	*Translate the question to a mathematical equation.*
$n \approx 0.662$	*Solve the equation for n.*
$n \approx 66.2\%$	*Write the decimal as a %.*

Once again, 45 successful free throws is approximately 66.2% of 68 attempts.

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Now You Try (Section 7)

Answer the following questions, using either method.

1) 40% of 170 is what number?

2) What number is $5\frac{1}{4}$ % of 2500?

3) 29% of what number is 34? (Round answer to the nearest tenth.)

4) 32 is 50% of what number?

5) What percent of 200 is 60?

6) If there are 346 students in the 8th grade at Keene Middle School and 198 are girls, what percent of the 8th grade is girls? (Round answer to the nearest tenth of a percent.)

(Answers to **Now You Try** (Section 7) *are found on page 20.)*

Section 8: Percent Application Problems

These problems are different from the ones in the last section in that you are not given the question explicitly. Instead, you need to write a question from the given word problem. Once you determine the question being asked, you use the same procedure as in section 7.

Example 1: About 65% of the students at a certain college receive some form of financial aid. How many of the 9280 students enrolled this year are receiving aid?
(Source: Prealgebra by Lial and Hestwood)

Solution: Using the general form, $\boxed{\%\ of\ total\ is\ part}$, we can write the question:

65% of 9280 total students is what number receiving aid?

Method 1:

$$\frac{65}{100} = \frac{n}{9280}$$

$$\frac{100n}{100} = \frac{603200}{100}$$

n = 6032

Method 2:

0.65 · 9280 = n

6032 = n

Therefore, 6032 students are receiving financial aid this year.

Example 2: A frozen dinner advertises that only 18% of its calories are from fat. If the dinner contains 55 calories from fat, what is the total number of calories in the dinner? Round the answer to the nearest whole number of calories. *(Source: Prealgebra by Lial and Hestwood)*

Solution: Using the general form, $\boxed{\%\ of\ total\ is\ part}$, we can write the question:

18% of what total calories is 55 calories from fat?

Method 1:

$$\frac{18}{100} = \frac{55}{n}$$

$$\frac{18n}{18} = \frac{5500}{18}$$

n ≈ 306

Method 2:

$$\frac{.18 \cdot n}{.18} = \frac{55}{.18}$$

n ≈ 306

Therefore, there are approximately 306 total calories in the frozen dinner.

Now You Try (Section 8)

Answer the following questions: *(Source: Prealgebra by Lial and Hestwood)*

1) There were 50 points in the first math test. Hue's score was 83% correct. How many points did Hue earn.

2) The rainfall in the Red River Valley was 33 inches this year. The average rainfall is 30 inches. This year's rainfall is what percent of the average rainfall?

3) A newspaper article stated that 648 pints of blood were donated at the blood bank last month, which was only 72% of the number of pints needed. How many pints of blood were needed?

(Answers to **Now You Try** (Section 8) *are found on page 20.)*

Exercises for Percents

Do all problems on a separate piece of paper, showing all work.

1. Write the following as an equivalent fraction without the % symbol.

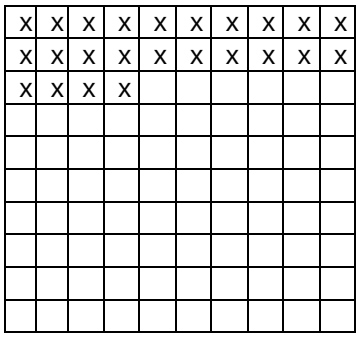
   a) 9% b) 17% c) 470%

2. Write the following using the % symbol.

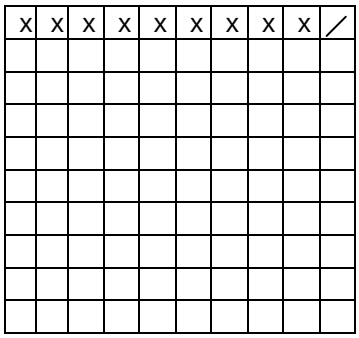
   a) $\frac{3}{100}$ b) $\frac{50\frac{1}{3}}{100}$

3. For each of the following Decimal Squares, determine how much is shaded. Write your answer as a percent, decimal, and a fraction.

a)



b)



4. Change the following percents to decimals.

   a) 91% b) 8% c) 0.11% d) 104%

5. Change the following decimals to percents.

   a) 0.85 b) 0.2 c) 0.0014 d) 7

6. Change the following percents to fractions.

a) 76% b) 67.3% c) $3\frac{3}{4}\%$

7. Change the following fractions to percents.

a) $\frac{7}{80}$ b) $\frac{9}{17}$

8. Complete the following chart with the correct common fraction, decimal, or percent equivalents.

	Common Fraction	Decimal	Percent
a)	_____	.5	_____
b)	1/3	_____	_____
c)	_____	_____	25%
d)	3/4	_____	_____
e)	_____	.2	_____
f)	_____	_____	40%
g)	4/5	_____	_____
h)	_____	.16666	_____
i)	_____	_____	$83\frac{1}{3}\%$

9. What number is 20% of 180?

10. What percent of 16 is 9?

11. 16 is 20% of what number?

12. 24 is what percent of 16?

13. 79% of 980 is what number?

14. How much HCL (hydrochloric acid) is in a 60-millimeter bottle that is marked 80% HCL?

15. There are about 55,000 words in Webster's Dictionary, but most educated people can identify only 20,000 of these words. What percent of the words in the dictionary can these people identify?
(Source: Prealgebra by Lial and Hestwood)

16. A survey at an intersection found that of 2200 drivers, 38% were wearing seat belts. How many drivers in the survey were wearing seat belts? *(Source: Prealgebra by Lial and Hestwood)*

17. So far this season, Kevin Garnett has made 76% of his free throws. He has made 133 free throws. How many has he attempted? *(Source: Prealgebra by Lial and Hestwood)*

Answers to Now You Try

Section 1:

1. a) $\frac{5}{100}$ b) $\frac{7.6}{100}$ c) $\frac{270}{100}$

2. a) 16% b) $9\frac{1}{8}\%$

Section 2:

1. a) i) 4% ii) $\frac{4}{100}=\frac{1}{25}$ iii) 0.04

   b) i) $21\frac{1}{2}\%$ ii) $\frac{21.5}{100}=\frac{215}{1000}=\frac{43}{200}$ iii) 0.215

2. a) i) shade 15 and $\frac{1}{2}$ squares

   ii) since $15\frac{1}{2}\%$ = 15.5%, then 15.5% = $\frac{15.5}{100}=\frac{155}{1000}=\frac{31}{200}$

   iii) $\frac{155}{1000}$ = .155

   b) i) shade 6 and $\frac{1}{4}$ squares

   ii) since $6\frac{1}{4}\%$ = 6.25%, then 6.25% $\frac{6.25}{100}=\frac{625}{10000}=\frac{1}{16}$

   iii) $\frac{625}{10000}$ = .0625.

3. Draw another Decimal Square. Then shade the entire given Decimal Square plus one column of the second Decimal Square you drew.

Section 3:

1. 0.98 2. 0.01 3. 0.1007 4. 0.009 5. 1.50

Section 4:

1. 67% 2. .07% 3. 80% 4. 120%

Section 5:

1. $\frac{13}{20}$ 2. 1 3. $\frac{12}{5}$ *or* $2\frac{2}{5}$

4. $\frac{173}{500}$ 5. $\frac{11}{150}$ 6. $\frac{1}{200}$

Section 6:

1. 75% 2. $42\frac{1}{2}\%$ or 42.5% 3. $41\frac{2}{3}\%$ or $41.\overline{6}\%$

Section 7:

1. 68 2. 131.25 3. 117.2

4. 64 5. 30% 6. 57.2%

Section 8:

1. Hue earned 41.5, or $41\frac{1}{2}$, points. 2. This year's rainfall is 110% of the average rainfall.

3. 900 pints of blood were needed.