Math 171 Proficiency Packet on Decimals

Section 1: Introduction

The name and the place value for the first seven columns in our number system is as follows:

Millions Column	Hundred Thousands Column	Ten Thousands Column	Thousands Column	Hundreds Column	Tens Column	Ones Column
1,000,000	100,000	10,000	1,000	100	10	1

As we move from right to left, multiply by 10 each time. The value of each column is 10 times the value of the column on its right, with the rightmost column being 1. To understand the idea behind decimal numbers, we notice that moving in the opposite direction, from left to right, we divide by 10 each time.

If we are at the ones column and go to the right the next column would have to be

$$1 \div 10 = \frac{1}{10}$$ Ten**th**s

The next one after that will be

$$\frac{1}{10} \div 10 = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$$ Hundred**th**s

After that, we have

$$\frac{1}{100} \div 10 = \frac{1}{100} \cdot \frac{1}{10} = \frac{1}{1000}$$ Thousand**th**s

We could continue this process of dividing by 10 to move one column to the right as long as we wanted to. A **decimal point** is used to show where the ones column is. The decimal point is placed between the ones column and the tenths column.

Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths	Millionths
1,000	100	10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$	$\frac{1}{1000000}$

↑ Decimal Point

Example 1: Give the place value of each digit in the number 15.7236.

Solution: Starting with them leftmost digit , we have: 1 is in the tens column, 5 is in the ones column, 7 is in the tenths column, 2 is in the hundredths column, 3 is in the thousandths column, and 6 is in the ten-thousandths column.

Example 2: Write each number in words.

a) 0.3 b) 0.03 c) 0.003

Solution: a) 0.3 is "three tenths".

b) 0.03 is "three hundredths".

c) 0.003 is "three thousandths".

When a decimal contains digits to the left of the decimal point, the word "and" is used to indicate where the decimal point is when writing the number in words.

Example 3: Write the following in words.

a) 4.2 b) 8.027 c) 14.0215

Solution: a) 4.2 is "four and two tenths".

b) 8.027 is "eight and twenty-seven thousandths".

c) 14.0215 is "fourteen and $\underbrace{two\ hundred\ fifteen}$ $\underbrace{ten-thousandths}$".

↑ whole # part ↑ decimal pt. ↑ write as if it were a whole number ↑ place value name of the last digit

Example 4: Write each in decimal notation.

a) five and sixteen hundredths

b) one hundred nineteen and forty-two ten-thousandths.

Solution: a)
1. Write the whole # part: 5
2. Write the decimal point when read "and": 5.
3. Put dashes to represent the # of place values we need: 5. _ _ *(2 spaces needed for hundredths)*
4. Place the number after the "and" in the blanks, working from the right and moving to the left: 5. 1 6
5. If there are any blanks not filled in, fill in with zeros.

b) 119._ _ _ _
119. _ _ 4 2
119. 0 0 4 2

Example 5: Write each as a fraction or a mixed number. Do not simplify.

a) 0.003 b) 15.6547

Solution: a) Since 0.003 is 3 thousandths, we write

$$0.003 = \frac{3}{1000}$$

b) Since 15.6547 is read "fifteen and six thousand five hundred forty-seven ten-thousandths", we write

$$15.6547 = 15\frac{6547}{10000}$$

Notice the whole number part stays a whole number, but the decimal part gets written as a fraction, with the place value name as the denominator.

Example 6: Write the following decimal numbers in order from smallest to largest.

6.02, 6.24, 6.0024, 6.204, 6.04

Solution: 1. Line up decimal points.

6.02
6.24
6.0024
6.204
6.04

2. Fill in with zeros so that all decimals have the same number of places.

6.0200
6.2400
6.0024
6.2040
6.0400

3. Since all the ones columns contain a 6, we ignore that column and treat the decimal portions as if they were whole numbers. Then compare those numbers, putting them in order from smallest to largest: 24, 200, 400, 2040, and 2400.

4. Write the decimals in order: 6.0024, 6.02, 6.04, 6.204, 6.24.

****

Now You Try (Section 1)

1. Give the place value of the digit 3 in the following numbers.

a) 193.876 b) 0.00432 c) 8.35

2. Write out the name of each of the numbers in words.

a) 1.73 _____

b) 109.005 _____

3. Write each number in decimal notation.

a) Eight and ninety-three thousandths _____

b) Sixteen and one hundred nine ten-thousandths _____

4. Write each number as a fraction or a mixed number. If possible, write answer in lowest terms.

a) 9.019 _____

b) 78.143 _____

c) 0.45 _____

5. Write the following decimal numbers in order from smallest to largest.

0.05 0.04 0.045 0.004 0.0405

(Answers to **Now You Try** (Section 1) *are found on page 16.)*

****

Section 2: A Model for Decimals

The following model for decimals is meant to give us a way to visualize decimals.

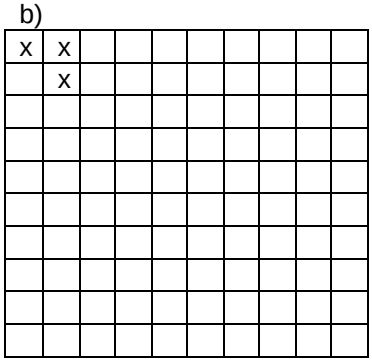
Hundreds Square. The hundreds square represents **1**. It is subdivided into ten rows of ten equal squares. Any row or column can be shaded to represent one **tenth**. Any of the small squares can be shaded to represent one **hundredth**.

Example 1: What number do the following figures represent?

a)

x	x	x							
x	x	x							
x	x	x							
x	x	x							
x	x	x							
x	x	x							
x	x	x							
x	x								
x	x								
x	x								

b)



Solution: a) Each column represents one tenth and there are two columns shaded, the two columns represent "two tenths". Each single square that is shaded represents one hundredth, so the seven shaded squares represent "seven hundredths". his would be written as $\frac{2}{10}+\frac{7}{100}$ or as a decimal, 0.27 .

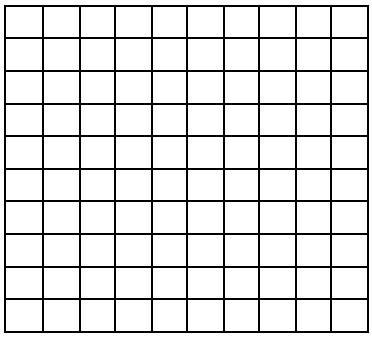
b) There are no columns or rows shaded so we have 0 tenths, $\frac{0}{10}$. The 3 shaded squares represent three hundredths, $\frac{3}{100}$. Hence the shaded portion of the square written as a decimal is $\boxed{0.03}$.

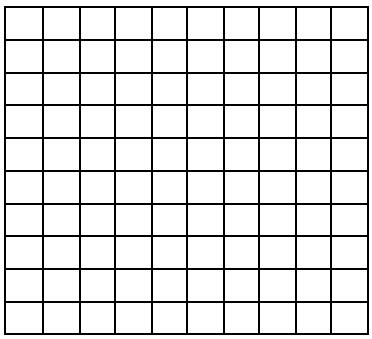
****

Now You Try (Section 2)

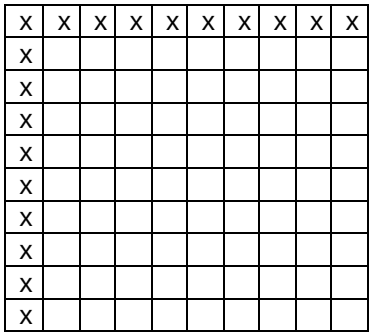
1. Shade the squares below to represent

a) 0.35 b) 0.09





2. The shaded portion of the following square represents what decimal?



(Answers to **Now You Try** (Section 2) *are found on page 16.)*

****

Section 3: Adding and Subtracting Decimals

The algorithm for **adding and subtracting decimals** is a three step process:

1. Line up decimal points. (Add zeros if necessary.)
2. Add, or subtract, ignoring decimal points.
3. Bring down the decimal point into the answer, directly below the decimal points in the problem.

Example 1: Add: 2.345 + 45.5.

Solution: 2.345 + 45.5 →

$$\begin{array}{r} 2.345 \\ +\ 45.5\mathbf{00} \\ \hline \downarrow \\ \boxed{47.845} \end{array}$$

Line up decimal points.
Add zeros.
Add numbers, bring decimal point down into the answer.

Example 2: Add: 5 + 1.0034.

Solution:

$$\begin{array}{r} 5.0000 \\ +\ 1.0034 \\ \hline \boxed{6.0034} \end{array}$$

Line up decimal points.
Add zeros.
Add numbers, bring decimal point down into the answer.

Example 3: Subtract: 27.87 - 11.654.

Solution: As with addition, we will first line up the numbers vertically, with the decimal points lined up, add zeros and then subtract as usual. Remember to place the decimal point in the answer directly below the decimal points in the problem.

$$\begin{array}{r} 27.870 \\ -\ 11.654 \\ \hline 16.216 \end{array}$$

Example 4: Subtract: 9 - 0.18.

Solution:

$$\begin{array}{r} 9.00 \\ -\ .18 \\ \hline \boxed{8.82} \end{array}$$

****

Now You Try (Section 3.1)

1. Add the following decimals.

   a) 0.14 + 7.6 b) 9.672 + 0.89 c) 18.7 + 0.93 + 6

2. Subtract the following decimals.

   a) 17.4 - 2.86 b) 8 - 0.53 c) 25.72 - 14.944

(Answers to **Now You Try** (Section 3.1) *are found on page 16.)*

****

Now let's look at some examples of adding and subtracting positive and negative decimals.

Example 1: Add: 4.75 + (-7.23).

Solution: Recall that to add two numbers with different signs, we have to subtract the smaller absolute value from the larger. The sign of the answer is the same as the sign of the number with the larger absolute value.

$|4.75| = 4.75$ and $|-7.23| = 7.23$

Now subtract the smaller absolute value from the larger absolute value.

$$\begin{array}{r} 7.23 \\ -\ 4.75 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} 7.23 \\ -\ 4.75 \\ \hline 2.48 \end{array}$$

Since 7.23 is larger than 4.75, we keep the sign of 7.23.

Therefore, 4.75 + (-7.23) = $\boxed{-2.48}$.

Example 2: Subtract: -5 - 2.34.

Solution: -5 - 2.34 = -5 + (-2.34) *Recall that subtraction = addition of the opposite.*

To add two numbers of the same sign, we add their absolute values and keep the sign of the original numbers.

-5 - 2.34 = $\boxed{-7.34}$

↑

The answer has the same sign as the original two numbers.

$$\begin{array}{r} 5.00 \\ +\ 2.34 \\ \hline 7.34 \end{array}$$

Add the absolute values of the two numbers.

Now You Try (Section 3.2)

1. Add the following decimals.

   a) -3.73 + 6.67 b) -4.8 + -17.03

2. Subtract the following decimals

   a) -9.67 - 14 b) 8.05 - 12.7

(Answers to **Now You Try** (Section 3.2) *are found on page 16.)*

****

Section 4: Multiplying Decimals

Like the algorithm for adding and subtracting decimals, the multiplication algorithm also involves three steps.

1. Multiply, ignoring decimal points.
2. Add up the number of decimal places in the factors.
3. Insert a decimal point in the product so that

$$\begin{bmatrix}\text{Number of}\\ \text{decimal places}\\ \text{in product}\end{bmatrix} = \begin{bmatrix}\text{Number of}\\ \text{decimal places}\\ \text{in first factor}\end{bmatrix} + \begin{bmatrix}\text{Number of}\\ \text{decimal places}\\ \text{in second factor}\end{bmatrix}$$

Example 1: Multiply: 1.751 x 2.34.

Solution:

```
  1.751     3 decimal places..
x 2.34      2 decimal places.
  7004      Multiply as usual.
 52530
350200
4.09734
```

The decimal point is placed so there are 3 + 2 = 5 decimal places.

Therefore, 1.751 x 2.34 = 4.09734.

Example 2: Multiply: 0.45 x 0.065.

Solution:

```
  0.45      3 decimal places.
x 0.065     2 decimal places.
   225      Multiply as usual.
  2700
0.02925
```

Since we need 2 + 3 or 5 decimal places in our answer, we need to add a zero before the first 2.

Therefore, 0.45 x 0.065 = 0.02925.

Now You Try (Section 4)

Multiply the following decimals.

1) 1.4 x 0.17 2) 4.65 x 2.67

3) 0.8 x 0.04 4) 2.604 x 14.03

(Answers to **Now You Try** (Section 4) *are found on page 16.)*

****

Section 5: Dividing Decimals

Division of decimals closely follows division of whole numbers. Suppose you had to find 123.45 ÷ 2.5. The traditional division algorithm for decimals is as follows:

1. Set up the calculation in the usual whole number format.

$2.5\overline{)123.45}$ *Recall the terms used in division:* $\begin{array}{r} quotient \\ divisor\overline{)dividend} \end{array}$

2. Move the decimal point in the divisor just enough places to the right so that the new divisor is a whole number, and move the decimal point in the dividend the same number of places to the right:

$25\overline{)1234.5}$

3. Divide as usual, and insert the decimal point in the quotient directly above the decimal point in the (new) dividend:

$$\begin{array}{r} 49.38 \\ 25\overline{)1234.50} \\ \underline{100} \\ 234 \\ \underline{225} \\ 95 \\ \underline{75} \\ 200 \\ \underline{200} \\ 0 \end{array}$$

← *Notice a zero was added to the dividend to continue the division.*

← *If a remainder of zero was not found, more zeros could be added to the dividend until a remainder of zero was found or instructions were given to round to a specific place value.*

Therefore, 123.45 ÷ 2.5 = $\boxed{49.38}$.

Example 1: Divide: 1 by 62.5.

Solution: Step 1: $62.5\overline{)1.}$ *Set up the calculation in the usual whole number format.*

Step 2: $625\overline{)10}$ *Move the decimal point in the divisor just enough places to the right so that the new divisor is a whole number, and move the decimal point in the dividend the same number of places to the right.*

Step 3:
$$\begin{array}{r} .016 \\ 625\overline{)10.000} \\ \underline{625} \\ 3750 \\ \underline{3750} \end{array}$$
Divide as usual, and insert the decimal point in the quotient directly above the decimal point in the (new) dividend.

Therefore, 1 ÷ 62.5 = 0.016.

****

Now You Try (Section 5)

Divide the following decimals.

1) 17.759 ÷ 3.01 2) 1.53 ÷ 7.5

(Answers to **Now You Try** (Section 5) *are found on page 16.)*

****

Section 6: Changing Fractions to Decimals

A fraction can be thought of as a numerator divided by a denominator. For example, the fraction $\frac{3}{5}$ can be thought of as 3 divided into 5 equal parts, or simply as 3 divided by 5. So to change a fraction to a decimal you just have to carry out the division.

Example 1: Change $\frac{3}{5}$ to a decimal.

Solution: Dividing 3 by 5 we get

$$\begin{array}{r} .6 \\ 5\overline{)3.0} \\ \underline{30} \\ 0 \end{array}$$

Therefore, $\frac{3}{5}$ = 0.6.

Example 2: Change $\frac{7}{8}$ to a decimal.

Solution: We have to divide 7 by 8. It is always a good idea to have an estimate of the answer before we begin. Since $\frac{7}{8}$ is greater than $\frac{1}{2}$ and less than 1, the answer has to be between .5 and 1.

$$
\begin{array}{r}
.875 \\
8\overline{)7.000} \\
\underline{64} \\
60 \\
\underline{56} \\
40 \\
\underline{40} \\
0
\end{array}
$$

Therefore, $\frac{7}{8}$ = $\boxed{.875}$.

Many times when changing a fraction to a decimal, the decimal does not terminate like the last two examples. For example, $\frac{1}{3}$ = .333333…, where the three dots mean that the decimal never terminates. Another way to write this is $\frac{1}{3} = .\overline{3}$ where the line goes above the part that repeats.

Example 3: Change $\frac{9}{11}$ to a decimal.

Solution: Dividing 9 by 11, we get

$$
\begin{array}{r}
.8181 \\
11\overline{)9.000} \\
\underline{88} \\
20 \\
\underline{11} \\
90 \\
\underline{88} \\
20
\end{array}
$$

Notice that both the 8 and 1 repeat.

Therefore, $\frac{9}{11}$ = $\boxed{.\overline{81}}$.

Now You Try (Section 6)

Change the following fractions to decimals.

1) $\frac{7}{20}$

2) $\frac{1}{40}$

3) $\frac{5}{6}$

4) $\frac{3}{11}$

(Answers to **Now You Try** (Section 6) *are found on page 16.)*

****

Section 7: Rounding Decimals

The rule for rounding decimal numbers is similar to the rule for rounding whole numbers.

1. If the digit in the column to the right of the one you are rounding to is 5 or more, add 1 to the digit in the column you are rounding to; otherwise, you leave it alone.

2. Replace all digits to the right of the column you are rounding to with zeros if they are to the left of the decimal point; otherwise, you delete them.

Example 1: Round: 467.999 to the nearest ten.

Solution: The number next to the tens column is 7, which is 5 or more, so you add 1 to 6. You change all digits to the right of the tens place to 0, and drop all digits after the decimal point.

467.999 ≈ 470.

Example 2: Round: 1.006349 to the nearest ten-thousandth.

Solution: Since the number to the right of the ten-thousandths column is 4, which is less than 5, the 3 is left alone. All digits to the right of 3 are deleted.

1.006349 ≈ 1.0063.

Example 3: Round: 143.9578 to the nearest tenth.

Solution: Since the number to the right of the tenths column is 5, the 9 increases to 10, which increases the 3 in the ones column to a 4. Place a zero in the tenths column to hold that place, then drop all digits to the left of the tenths place.

Therefore, 143.9478 ≈ 144.0.

This zero is important to show that we rounded to the tenths place.

Now You Try (Section 7)

1) Complete the following table by rounding each decimal to the indicated places.

DECIMAL	WHOLE NUMBER	TENTH	HUNDREDTH
4.099			
4.936			
0.074999			
0.545			

2) Round 9.05496 to the nearest ten-thousandth.

3) Round 10,479.057 to the nearest thousand.

(Answers to **Now You Try** (Section 7) *are found on page 17.)*

Exercises for Decimals

Do all problems on a separate piece of paper, showing all work.

1. Write the name of each decimal in words.

   a) 0.103 b) 5.02 c) 24.6 d) 4.043

2. Write each number as a fraction or a mixed number. Do not simplify your answer.

   a) 2.96 b) 65.101 c) 0.00062

3. a) Give the place value of the 1 in each of the following numbers.

   i) 314.67 ii) 65.12 iii) 0.0021 iv) 8.54321

   b) Give the place value of the 9 in each of the following numbers.

   i) 56.902 ii) 153.3759 iii) 59.036

4. Write each of the following as a decimal.

   a) Six and two tenths b) Two hundred and 8 hundredths

   c) Five thousand and five thousandths

   d) Twelve thousand and four hundred thirty-five thousandths

   e) Seventeen and four hundred six ten-thousandths

5. Write the following numbers in order from smallest to largest.

   a) 0.03, 0.02, 0.025, 0.0099, 0.209

   b) 5.7, 5.07, 5.27, 5.027, 5.207

6. Change each decimal to a fraction, and then simplify.

   a) 0.35 b) 0.125 c) 0.0605

   d) 0.0500 e) 0.1742

7. Without a calculator, find the following.

   a) 4.56 + 2.09 b) 0.04 + 0.83 + 0.72

   c) 5.0004 + 2.97 + 0.008 d) 0.81 + (-5) + 4.95

   e) 65.0197 + 6.78 + 0.0009 f) 987.658 + 341.396

g) 76.56 - 49.82 h) 5.94 - 4.84

i) -5 - 0.943 j) -12 - 5.052

k) 45.238 - 21.753 l) 34.07 - 6.18

m) 50.05 - 5.5 n) -34.7 - (-7.62)

o) -5.78 + 4.35 p) 411.4 ÷ 44

q) 2.3 x 4.52 r) 21.978 ÷ 3.3

s) 2.40 ÷ 0.75 t) 4.005 x 0.97

u) 372 ÷ 80 v) 1.068 ÷ 0.012

w) 7.26 x 2.7

8. What number is added to 0.58 to obtain 7.34 ?

9. Carry out each of the following divisions only so far as needed to round the results to the nearest hundredth.

a) 4 ÷ 7 b) 0.0743 ÷ 0.49

10. Write each fraction as a decimal. Round to the nearest thousandth if necessary.

a) $\frac{4}{5}$ b) $\frac{13}{60}$ c) $\frac{9}{80}$ d) $\frac{3}{7}$

11. Round the decimal to the given place value.

a) 0.6235; nearest hundredth b) 15.8479; nearest thousandth

c) 109.543; nearest ones d) 16.34921; nearest ten-thousandth

Answers to Now You Try

Section 1:

1) a) ones b) ten-thousandths c) tenths

2) a) one and seventy-three hundredths

b) one hundred nine and five thousandths

3) a) 8.093 b) 16.0109

4) a) $9\frac{19}{1000}$ b) $78\frac{143}{1000}$ c) $\frac{45}{100}=\frac{9}{20}$

5) 0.004, 0.04, 0.0405, 0.045, 0.05

Section 2:

1) a) shade 3 columns and 5 squares b) shade 9 squares

2) 0.19 *(Be careful not to count the corner square twice.)*

Section 3.1:

1) a) 7.74 b) 10.562 c) 25.63

2) a) 14.54 b) 7.47 c) 10.776

Section 3.2:

1) a) 2.94 b) -21.83

2) a) -23.67 b) -4.65

Section 4:

1) 0.238 2) 12.4155 3) 0.032 4) 36.53412

Section 5:

1) 5.9 2) 0.204

Section 6:

1) 0.35 2) 0.025 3) $0.8\overline{3}$ 4) $0.\overline{27}$

Section 7:

1)

DECIMAL	WHOLE NUMBER	TENTH	HUNDREDTH
4.099	4	4.1	4.10
4.936	5	4.9	4.94
0.074999	0	0.1	0.07
0.545	1	0.5	0.55

2) 9.0550

3) 10,000