Math 171 Proficiency Packet

<u>Algebra</u>

Math 171 Proficiency Packet on Algebra

Section 1: Algebraic Expressions

Consider the following problem:

Find a pattern and write the next three terms of the sequence 5, 8, 11, 14, ___, ___,

To get the next number in the sequence, you would add 3 to 14. We can get any term in the sequence by adding 3 to the preceding term. As long as we have a list of numbers we can get the next term, but what if we wanted to jump directly to the 200th term? One of the many uses of algebra is finding and describing a pattern.

To help us to describe a pattern using algebra it is useful to set up an input-output table. The input-output table for this problem would be

Number of term	1	2	3	4	•••	
(Input)						
Term (output)	5	8	11	14	•••	

We want to find a way the input relates to the output, in other words, how the number of a term relates to the term itself. Looking at the table above, we notice the following

Number of term (Input)	1	2	3	4		
Term (output)	5	5+3	5+3+3	5+3+3+ 3	•••	

We start with 5, to get the second term we add one 3, to get the third term we add two 3's, and to get the fourth term we add three 3's. Notice the number of 3's we add is always 1 less than the term. Now if we let n represent any input number and translate this pattern, the output will be 5+3(n-1).

There are a number of ways to write multiplication: 3(n-1), $3\cdot(n-1)$, or (3)(n-1). It should be noted that $3 \times (n-1)$ is never used because of the confusion as to whether x represents the multiplication operation or a letter to be multiplied.

Another column is added to our input-output table and we write the expression 5+3(n-1) the output for n.

Number of term (Input)	1	2	3	4	 n
Term (output)	5	8	11	14	 5+3(n-1)

There are some very important ideas used here which we will define on the next page.

Definition: A variable is a letter or symbol that represents any number in the input(or output) set.

The letter n was the input variable. Any number appropriate to the input set may replace the variable. In our example, the only numbers appropriate for n are the counting (natural) numbers. It would not make sense to talk about the $1\frac{1}{2}$ term, so n could not equal $1\frac{1}{2}$.

Definition: A **constant** is a quantity whose value does not change.

In the expression 5 + 3(n-1), the 5, 3, and 1 are constants.

An **algebraic expression** contains a combination of variables, numbers, and signs of operation. Algebraic expressions are mathematical phrases that can be combined to form mathematical sentences called **equations**. When you are working with expressions it is important to be able to determine the **terms** of an expression. Terms are part of an expression that are separated by addition or subtraction.

Example 1:

- a) 3n has only one term.
- b) $5x \text{ and } 3ab^2$ are the terms of the expression $5x + 3ab^2$.
- c) $3x^2y^5$ has only one term.
- d) x 7 has two terms, x and 7.

In the above example, $3x^2y^5$ has only one term, however this one term it is made up of three **factors**. Factors are numbers (**coefficients**) or variables that are multiplied in a product. In this example the factors are 3, x^2 , and y^5 . In this example, 3 is also called a coefficient.

Now You Try (Section 1)

In each of the following algebraic expressions, name the terms and factors.

		number of terms	terms	factors
1)	Х			
2)	6x ²	1	6x ²	6 and x^2
3)	6x + 1			
4)	x(x + 1)			
5)	$x^2 + 4x - 9$			

(Answers to Now You Try (Section 1) are found on page 18.)

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Section 2: Translating Expressions

It is important to be able to translate from mathematical symbols into words and vice versa. For example, 3 + 4(3 + 12), would be translated into words as, 3 added to the product of 4 and the sum of 3 and 12. There are key words that help in the translation:

addition translates to words such as "increased by", "more", or 'sum"
subtraction translates to "less than", "decreased by", or "difference"
multiplication translates to "times" or "product"
division translates to "quotient" or "divided by"

In order to translate expressions, you need to keep in mind the order of operations, which are briefly reviewed below.

ORDER OF OPERATIONS

If **no** grouping symbols are present do the following:

- 1. Evaluate all powers, working from left to right.
- 2. Do any multiplications and divisions in the **order** in which they occur, working from left to right.
- 3. Do any additions and subtractions in the **order** in which they occur, working from left to right.

If grouping symbols are present, do the following:

- 1. First use the above steps within each pair of parentheses.
- 2. If the expression includes nested grouping symbols, evaluate the expression in the innermost set of parentheses first.

Grouping symbols may be (), [], or {}.

Note: Some people use the acronym "Please Excuse My Dear Aunt Sally" to remember the order of operations. Here the P stands for parentheses, E for exponent, M for multiplication, D for division, A for addition, and S for subtraction. This acronym can be very misleading for people because it can be misinterpreted to mean that multiplication must be performed before division and addition before subtraction. Remember, you do multiplication and division the order in which they appear, if a division appears before a multiplication you do division first. Similarly, if a subtraction appears before an addition, you do subtraction first.

Example 1: Evaluate 6 + 3(8 - 3) without using a calculator.

Solution: According to the order of operations, we have to calculate what is inside the grouping symbols first:

6 + 3(8 - 3) = 6 + 3(5) subtract 3 from 8 = 6 + 15 multiply 3 and 5 = 21 add 6 and 15

Example 2:

Translate the following English expressions into mathematical expressions written in symbols.

- a) 6 times the sum of 7 and 10
- b) The sum of 3 times 2 and 12 times 5
- c) Three times the difference of 14 and 8
- d) 6 subtracted from the quotient of 24 and 8

Solution:

a)

- b) $3 \cdot 2 + 12 \cdot 5$ \rightarrow Here we do not need the () because the order of operations tells us to multiply first, then add.
- c) 3(14 8) \rightarrow Note that the words "difference of" required the use of () so that we would subtract before multiplying.
- d) $24 \div 8 6 \rightarrow We \ can \ use \ either \ a \ fraction \ bar \ or \ \frac{24}{8} 6 \qquad quotient.$

Example 3:

Translate the following mathematical expressions into English expressions.

a) $(12+6) \div 3$

b) $7 + 8 \cdot 4$

Solution:

- a) The sum of 12 and 6 divided by 3
- b) 7 plus the product of 8 and 4

Now You Try (Section 2)

- Translate the following English expressions into mathematical expressions written in symbols.
 - a) 8 times the sum of 4 and 2.

- b) 4 added to 3 times the sum of 5 and 8
- c) The difference of 6 times 9 and the sum of 4 and 7.
- _____

2) Translate the following mathematical expressions into English expressions.

a) 30 - (19 - 5)

b) 4· 6+7

c) (4 + 6) ÷ 8

d) $\frac{4.6}{6-9}$

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Section 3: Exponents

Just as multiplication can be thought of as a shorthand notation for repeated addition, $3 \cdot 4 = 4 + 4 + 4$, exponents are a shorthand notation for repeated multiplication, $2^3 = 2 \cdot 2 \cdot 2$. So **exponential notation** is just a shorthand system, but like any special notation you have to learn to read and use it correctly.

Exponential Notation Let x be a real number and n be a positive integer. The product of n of the x's

is called

"x to the nth power" or "x to the nth" or "x raised to the n."

We call n the **exponent** or "power" and x the **base**, and write

When dealing with exponential notation it is very important to determine what the base of an exponent is. As a general rule, the exponent applies to the symbol that precedes it. For example, the symbol that precedes the exponent in 5^3 is 5, so the base for the exponent is 5. This means that 5 is used as a factor 3 times, $5^3 = (5)(5)(5)$. Now consider $(4x)^2$, here the symbol that precedes the exponent is parentheses. This tells us that everything inside the parentheses is the base, so here 4x will be used as a factor twice. In other words, $(4x)^2 = (4x)(4x)$. Remember that when parentheses are used, it is the entire quantity inside the parentheses that is repeated as the factor.

Now You Try (Section 3.1)

Name the base for each exponent.

3)
$$(x + y)^2$$

4)
$$4x^3$$

(Answers to Now You Try (Section 3.1) are found on page 18.)

Rules and Properties of Exponents

Before continuing, we need to review how integer exponents work.

Rules of Exponents: For m and n any positive integers and $b \neq 0$:

1.
$$b^{m} \cdot b^{n} = b^{m+n}$$

$$2. \qquad \frac{b^{m}}{b^{n}} = b^{m-n}$$

3.
$$(b^{m})^{n} = b^{mn}$$

4.
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

5.
$$(ab)^n = a^n b^n$$

Example 1: Simplify the following:

a)
$$y^5 \cdot y^7 \cdot y$$
 b) $(5y^3)^2$ c) $\frac{10y^5}{2y^3}$

b)
$$(5y^3)^2$$

c)
$$\frac{10y^5}{2y^3}$$

Solution:

a)
$$y^4 \cdot y^7 \cdot y = y^{12}$$

property 1 for exponents (remember $y = y^1$)

b)
$$(5y^3)^2 = 5^2(y^3)^2$$

= $25y^6$

property 5 for exponents property 3 for exponents

c)
$$\frac{10y^5}{2y^3} = \frac{10}{2} \cdot \frac{y^5}{y^2}$$

separate product into 2 factors

$$=5y^3$$

divide 10 by 2 and property 2 of exponents

Now You Try (Section 3.2)

Simplify the following:

1)
$$3x^3 \cdot 7x^5 =$$

2)
$$(x^7)^3 =$$

3)
$$(3x^7)^3 =$$

4)
$$\frac{27x^7}{3x^2} =$$

(Answers to Now You Try (Section 3.2) are found on page 18.)

Section 4: Simplifying Algebraic Expressions

When we are asked to simplify an algebraic expression, we are basically being asked to make it as compact as possible by using the distributive property, combining like terms, and following the order of operations.

In order to simplify algebraic expressions, we need to be able to combine like terms. Recall that like terms are terms that have the same variable raised to the same degree. For example, x and 3x are like terms,

but $2x^2$ and 5x are not like terms. Only like terms can be combined.

Example 1: Simplify: 12x + 7 - 4x

Solution: Since 12x and - 4x are like terms, we can combine them to get 8x, giving us

$$12x+7-4x=8x+7$$

We also need to recall the distributive property to simplify algebraic expressions. The numerical expression 4(6 + 7) can be computed two ways:

- 1. using the order of operations, 4(6 + 7) = 4(13) = 52,
- 2. using the **distributive property**, which states a(b + c) = ab + ac.

The distributive property states that the product of one number and the sum of two or more other numbers is equal to the sum of the products of the first number and each of the other numbers of the sum. For example using the distributive property on the expression 4(6 + 7), we get

$$4(6+7) = 4 \cdot 6 + 4 \cdot 7 = 24 + 28 = 524(6+7)$$

With numerical expressions, we have a choice of what to do, but if we have an algebraic expression within a set of parentheses, we would need the distributive property to simplify.

Example 2: Simplify: 8(2x + 5)

Solution: Since we cannot add 2x and 5, we would need to distribute the 8 to simplify.

$$8(2x+5) = 8 \cdot 2x + 8 \cdot 5 = 16x + 40$$

Example 3: Simplify: 4x - 2(x + 1)

Solution: Since we cannot add x and 1, we need to distribute the -2 that is in front of the

parentheses.

$$4x-2(x+1) = 4x-2x-2$$

Note: the negative in front of the 2 gets distributed as well, giving us -2x and -2.

Now, we can combine the like terms, 4x and -2x, to get

$$= 2x - 2$$

Example 4: Simplify: 12 + 5(3x - 6)

Solution: 12+5(3x-6)=12+15x-30 distribute the 5

=15x-18 combine like terms, 12 and -30

Now You Try (Section 4)

Simplify the following algebraic expressions.

- 1) 6(2x 7)
- -2(x + 15)
- 3) 6-2(3x-9)
- 4) 7(x-8) + 14x
- 5) $3(x^2 + 3x + 1) + 4(x + 5)$

(Answers to Now You Try (Section 4) are found on page 18.)

Section 5: Evaluating Algebraic Expressions

When we are asked to evaluate an algebraic expression, we are basically being asked to substitute the given value for the variable and then simplify the resulting expression.

Example 1: Evaluate:

a)
$$4x + 5y$$
 when $x = -2$ and $y = 4$

b)
$$6x^2$$
 when $x = -3$

c)
$$\frac{(x+y)^2}{12}$$
 when x = 2 and y = -8

Solution: a)
$$4(-2) + 5(4)$$
 Substitute -2 in for x and 4 in for y .
$$-8 + 20$$
 Multiply.
$$12$$
 Add.

b)
$$6(-3)^2$$
 Substitute -3 in for x.
 $6 \cdot 9$ Evaluate $(-3)^2 = (-3)(-3) = 9$.
 54 Multiply.

c)
$$\frac{(2+-8)^2}{12}$$
 Substitute 2 in for x and -8 in for y.
$$\frac{(-6)^2}{12}$$
 Combine what is in the parentheses.
$$\frac{36}{12}$$
 Evaluate $(-6)^2 = (-6)(-6) = 36$. Divide.

Now You Try (Section 5)

Evaluate the following algebraic expressions.

1)
$$3x - 1$$
 when $x = 6$

2)
$$8x^2$$
 when $x = -4$

3)
$$\frac{(x-y)^3}{25}$$
 when x = 2 and y = -3

(Answers to Now You Try (Section 5) are found on page 19.)

Section 6: Solving Equations

One of the basic goals of algebra is to solve equations. An **equation** is a mathematical statement in which two expressions equal one another. Manipulating expressions and equations is important because sometimes the form of an expression might be inappropriate for a particular task and you will have to simplify expressions or combine expressions to produce a new one.

In algebra we are often interested in finding **solutions** or **roots** of an equation. Two equations are **equivalent** if they have exactly the same roots. There is a series of operations that will allow you to change an equation into an equivalent equation.

Operations for Changing Equations into Equivalent Equations

- 1. Add or subtract the same number or algebraic expression to both sides of the equation.
- 2. Multiply or divide both sides by the same number. Note: You cannot divide both sides by 0.
- 3. Combine like terms on either side of the equation.
- 4. Interchange the two sides of the equation.

It is important to note that you add or subtract terms from both sides and you multiply or divide both sides by factors.

The following equations are equivalent because each has a solution or root of 5. Check this.

a)
$$4x = 20$$

b)
$$3x - 2 = 13$$

c)
$$x = 5$$

d)
$$\frac{3x-5}{2} = 2x - 5$$

e)
$$-3 = 2 - x$$

Our goal when solving an equation algebraically is to transform the equation into a simpler but equivalent equation. For example, when we are asked to solve an equation like $\frac{2x-5}{3} = 2x + 3$, we have to find an equivalent equation that is in the form x = "something". We accomplish this by using the four operations above.

Example 1: Solve 3x + 27 = 6x

Solution: 3x + 27 = 6x

Our goal is to obtain an equation in the form x = #.

27 = 3x Subtract 3x from both sides.

 $\frac{27}{3} = \frac{3x}{3}$ Divide both sides by 3.

9 = x or x = 9 Interchange the two sides of the equation.

Now let's check the solution by substituting a 9 for x in the original equation. Does 3(9) + 27 = 6(9)? Yes, it does since 27 + 27 = 54.

Example 2: Solve 7y + 6 = 216 - 3y

Solution: 7y + 6 = 216 - 3y Our goal is to obtain an equation in the form y = #.

10y + 6 = 216 Add 3y to both sides.

10y = 210 Subtract 6 from both sides.

y = 21 Divide both sides by 10.

Check the solution by substituting 21 for y in the original equation. 7(21) + 6 = 153 and 216 - 3(21) = 153, so y = 21 is the correct solution.

Contrary to what many people think, there is **not** only one "correct" way to solve a given equation. In the next example we will solve an equation in a non-standard way to illustrate the fact that there are many ways to solve an equation.

Example 3: Solve 3(2x - 6) = 24

Solution: a) We will do this problem first by eliminating the 3 from the left side. Three is a factor so we have to divide both sides by 3. Then to eliminate the 2 from the left side we have to divide both sides (every term) by 2.

3(2x - 6) = 24

2x - 6 = 8 Divide both sides by 3.

x - 3 = 4 Divide both sides (every term) by 2.

x = 7 Add 3 to both sides.

b) Now let's solve this equation using a more common approach.

3(2x - 6) = 24

6x - 18 = 24 Distribute the 3. 6x = 42 Add 18 to both sides.

x = 7 Divide both sides by 6.

Now You Try (Section 6)

Solve the following equations.

1) 6(x-2) + 12 = 30

2) 8 + 4(5x - 1) = 10

3) 6 - 2(3x - 2) = 7

4) -8x + 7 = 37 - 5x

(Answers to Now You Try (Section 6) are found on page 19.)

Section 7: Checking Solutions

As seen in section 6, once we solved an equation we checked to see if we obtained the correct solution. If we substitute the solution into the original equation, we must obtain a true solution. All the equations we have looked at have been linear equations in one variable, which means the single variable in the equation has been raised to the first degree.

We can also look at linear equations involving two variables. These linear equations produce a straight line when graphed and are modeled by the equation y = mx + b, where (x,y) is a point on the coordinate axes, m is the slope of the line, and b is the y-intercept. To determine if a given point is a solution to a linear equation in two variables, the same process is followed as done above, except values for both x and y must be substituted into the given equation. Once the values are substituted, the result should be a true statement for the point to be a solution.

Example 1: Is the point (3, -2) on the line y = 2x - 9?

Solution: To determine if (3, -2) is a point on the given line, substitute 3 for x and -2 for y.

$$-2 = 2(3) - 9$$

 $-2 = 6 - 9$
 $-2 \neq -3$

Since we did not get a true statement, the point (3, -2) is not a solution to y = 2x - 9 and thus is not a point on the line created by that equation.

Example 2: Is the point (-5, -9) on the line y = 3x + 6?

Solution: To determine if (-5, -9) is a point on the given line, substitute -5 for x and -9 for y.

$$-9 = 3(-5) + 6$$

 $-9 = -15 + 6$
 $-9 = -9$

Since we did get a true statement, the point (-5, -9) is a solution to y = 3x + 6 and thus is a point on the line created by that equation.

Now You Try (Section 7)

Determine if the following points are solutions to the given equations.

- 1) Is the point (2, -4) on the line y = 3x 7?
- 2) Is the point (-1, 1) on the line y = 7x + 8?
- 3) Is the point (-4, 8) on the line y = -5x 12?

(Answers to Now You Try (Section 7) are found on page 19.)

Section 8: Formulas

A **formula** is an equation that contains more than one variable. Formulas, which are also called **mathematical models**, appear in all fields from engineering to economics.

Evaluation of formulas requires the substitution of numerical values for the variables as we did in section 5, but with formulas, you have an equation, not just an algebraic expression.

Example 1: The formula for the distance driven is D = RT, where D = distance driven, R = RT

speed in miles per hour, and T = time in hours. Determine the distance driven

when R = 55 mph and T = 3 hours.

Solution: To determine the distance, substitute 55 for R and 3 for T in the formula, D = RT.

D = 55(3) = 165 miles

Now You Try (Section 8.1)

1) Given $A = \frac{1}{2}bh$, find A when b = 5 inches and h = 12 inches.

2) Given
$$F = \frac{9}{5}C + 32$$
, find F when $C = 30^{\circ}$.

(Answers to Now You Try (Section 8.1) are found on page 19.)

Often it is necessary to manipulate the formula to solve for a particular variable that appears in it. We will accomplish this by the same steps we used to solve an equation. The only difference is that instead of getting a numerical value, we will solve and end up with one variable equaling a variable expression.

Example 2: Solve P = 4L + 2W, for W.

Solution: Our goal is to get W = "something". To accomplish this we have to eliminate the term 4L and the factor 2 from the right side of the equation. First, isolate the variable expression containing W by subtracting 4L from both sides.

$$P = 4L + 2W$$

$$-4L - 4L$$

$$P - 4L = 2W$$

Next, we need to isolate W by dividing both sides by 2.

$$\frac{P-4L}{2}=W$$

Example 3: Solve
$$V = \frac{1}{3} pr^2 h$$
, for h.

$$3 \cdot V = 3 \cdot \frac{1}{3} pr^2 h$$
$$3V = pr^2 h$$

Next, to isolate the h, divide both sides of the equation by pr^2 .

$$\frac{3V}{pr^2} = \frac{p\kappa^2 h}{p\kappa^2}$$

$$\frac{3V}{pr^2} = h$$

So,
$$h = \frac{3V}{pr^2}$$
.

Now You Try (Section 8.2)

1) Solve for R:
$$D = RT$$

2) Solve for H:
$$V = LWH$$

3) Solve for
$$b_2$$
: $A = \frac{1}{2}(b_1 + b_2)h$

4) Solve for s:
$$M = n + 0.3s$$

(Answers to Now You Try (Section 8.2) are found on page 19.)

Exercises for Algebra

Do all the exercises on separate paper showing all your work.

- 1. In each of the following algebraic expressions, name the terms and factors.
 - a) 4x + 8
 - b) 5(x + 8)
 - 6 4(a + 7)c)
 - (4x + 7)(x 10)d)
- 2. Translate the following verbal phrases into mathematical expressions or sentences using variables.
 - a) Seven added to the product of six times a certain number.
 - b) Three less than the product of four times a number.
 - c) The quotient of three times a number and five.
- 3. Translate the following mathematical expressions into English expressions.
 - $15 + 9 \div 3$ a)
 - b) 6(11-4)
- 4. Name the base for the exponents in the following:
- a) $(3x)^2$ b) $4x^4$ c) $(x+6)^2$ d) $-4x^4$

- Perform the indicated operations. 5.

- a) 5^3 b) $x^6 \cdot x^8$ c) $(2y^3)^4$ d) -4^2 e) $(\frac{3x^3}{v^5})^3$ f) $(-x)^2 \cdot (x)^4$

- 6. Simplify the following algebraic expressions.
- a) 15x-12+9x b) 3x+4(x-1) c) 13-9(2x+4)
- d) 16(x-3) + 7x e) $6(2x^2 4x 14) + 3(5x 8)$
- 7. Evaluate:
- $\frac{5(F-32)}{9}$ when F = 104 b) $\frac{(x-y)^2}{9}$ when x = -2 and y = 4
 - 2(b-5)+7 when b=16c)
- d) $a^2 5a + 9$ when a = -3

- 8. Solve the following equations for x.
 - a) 4x 9 = 11
- b) 5 3(4 5x) = 23 c) 6x + 4 = 34 d) -3x 8 = 19

- e) 5(x-4)+7=3x+10
- f) 4(x-1)-3(x-4)=7
- 9. Determine if the following points are solutions to the given equations.
 - a) Is the point (4, -12) on the line y = -6x + 12?
 - b) Is the point (-3, -20) on the line y = 4x 9?
- a) Given $A = L \cdot W$, find the area if the length = $5\frac{1}{2}$ ft and the width = 4ft. 10.
 - b) Given $A = \frac{1}{2}(b_1 + b_2)h$, find A when $b_1 = 8$, $b_2 = 12$, and h = 5.
- A + B + C = 180a) Solve for B: 11.
 - A = 3M 2Nb) Solve for N:
 - $K = \frac{1}{2}mv^2$ c) Solve for m:

Answers to Now You Try

Section 1:

problem	expression	number of terms	terms	factors
#1	Х	1	х	Х
#2	6x ²	1	6x ²	6 and x^2
#3	6x + 1	2	6x and 1	(6x + 1)
#4	x(x + 1)	1	x(x + 1)	x and (x + 1)
#5	$x^2 + 4x - 9$	3	x^{2} , $4x$, and -9	$(x^2 + 4x - 9)$

Section 2:

- 1) a) 8(4+2)
- b) 4 + 3(5 + 8)
- c) $6 \bullet 9 (4+7)$ or $(6 \bullet 9) (4+7)$, but the () really aren't needed around $6 \bullet 9$ because the order of operations will dictate that be done first.
- a) thirty decreased by the difference of nineteen and five 2)
 - b) four times six plus seven
 - c) the sum of four and six divided by eight
 - d) the product of four and six divided by the difference of six and nine

Section 3.1:

- 1) -5
- 2) 5 (only the 5 is squared, the negative is <u>not</u> included.)
- 3) x + y
- 4)
 - x (only the x is cubed, not the 4.)

Section 3.2:

- $21x^{8}$ 1)
- 2)
- $27x^{21}$ 3)
- $9x^5$ 4)

Section 4:

- 12x 421)

 x^{21}

- 2) -2x-30 3) -6x+24
- 21x 564)

 $3x^2 + 13x + 23$ 5)

Section 5:

- 1) 17
- 2) 128 3) 5

Section 6:

- 1) x = 5 2) $x = \frac{3}{10}$ 3) $x = \frac{1}{2}$ 4) x = -10

Section 7:

- 1) no
- 2)
- yes 3) yes

Section 8.1:

- 1) A = 30 square inches 2) $F = 84^{\circ}$

Section 8.2:

- 1) $R = \frac{D}{T}$ 2) $H = \frac{V}{LW}$ 3) $b_2 = \frac{2A}{h} b_1$ 4) $s = \frac{M n}{0.3}$