Percents

Section 1: Introduction

Percentages are just another way of expressing fractions. **Percent** means "per hundred" or "divided by one hundred." For example, 25% is the same as 25 one-hundredths. In other words,

$$25\% = \frac{25}{100}$$

or written as a decimal

$$25\% = 0.25 .$$

The following examples illustrate the meaning of percents.

Example 1 :

a) $6\% = \dfrac{6}{100}$

b) $1.9\% = \dfrac{1.9}{100}$

c) $110\% = \dfrac{110}{100}$

d) $45\frac{1}{3}\% = \dfrac{45\frac{1}{3}}{100}$ ♦

You can see that percents are just fractions (or ratios) with denominator 100.

Activity 1

Write the following as an equivalent fraction without the % symbol.

a)   5%   _____

b)   7.6%   _____

c)   270%   _____

Write the following using the % symbol.

d) $\dfrac{16}{100}$ _____

e) $\dfrac{7\frac{1}{5}}{100}$ _____

Section 2:   Percents Using Decimal Squares

Using Decimal Squares, it is easy to relate percents to fractions and decimals. The following grid is a Decimal Square; it is divided into 100 equal parts, and 30 parts are shaded. Since 30 out of 100 are shaded, we can say that 30% is shaded. The shaded portion also represents the decimal .30 and the fraction $\frac{30}{100}$ or $\frac{3}{10}$.
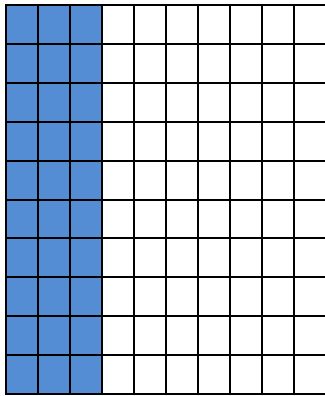
Example 2 :   Use the Decimal Square to shade $8\frac{1}{2}$ %.
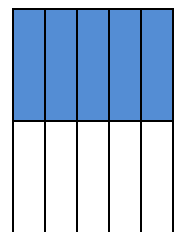
Solution: $8\frac{1}{2}$ % means $8\frac{1}{2}$ parts out of 100 equal parts, so we have to shade $8\frac{1}{2}$ of the squares.
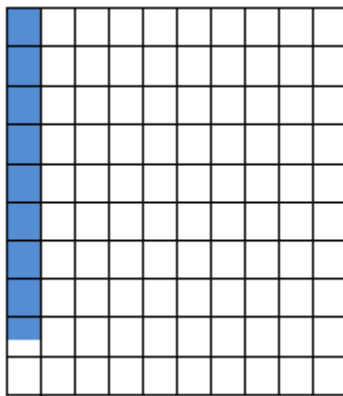
The square that is half shaded is shown to the right, divided into tenths.

If all the squares in the Decimal Square were divided in the same way, we would have 1000 equal parts and 85 would be shaded. Hence, $8\frac{1}{2}$ % is equivalent to $\frac{85}{1000}$ or 0.085.

Activity 2

For each of the following Decimal Squares, determine how much is shaded.
Write your answer as a percent, decimal, and a fraction.

a)
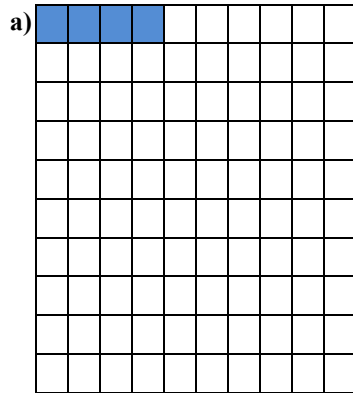
Percent   ____
Decimal   ____
Fraction   ____

Percent   ____
Decimal   ____
Fraction   ____

c)
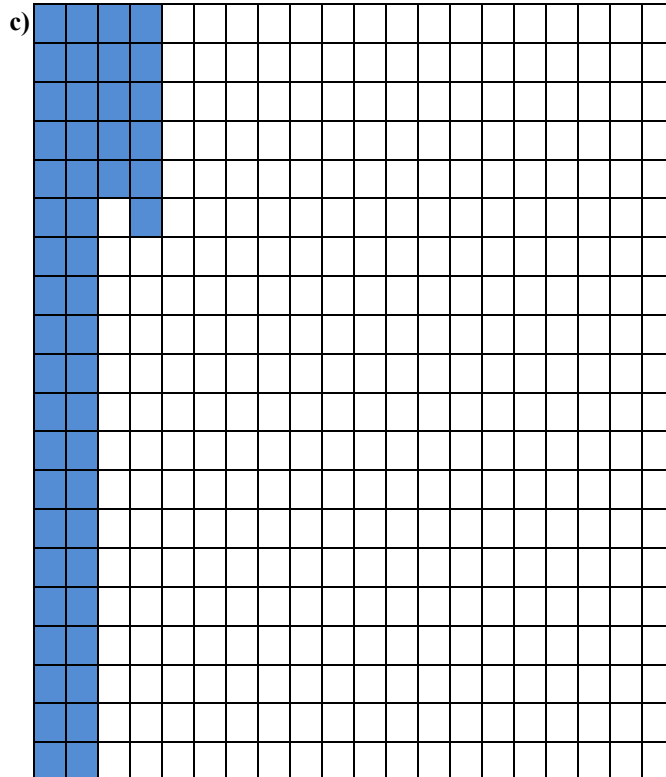
Percent   ____
Decimal   ____
Fraction   ____

Shade the indicated percent. Write the percent as a decimal and as a fraction.

a)

b)

$15\frac{1}{2}$ % $6\frac{1}{4}$ %
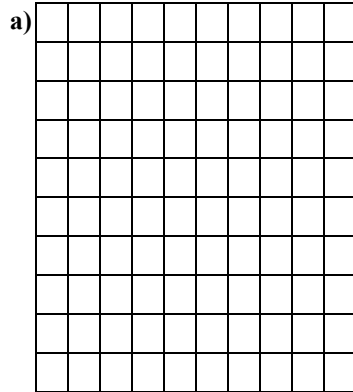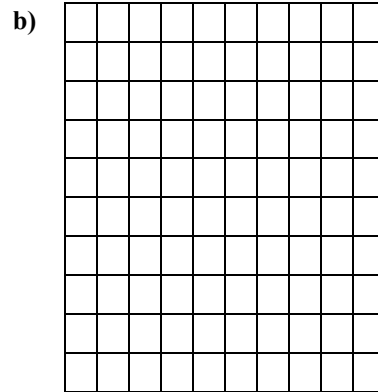
decimal ____   **decimal** ____

fraction ____   **fraction** ____

If you were going to shade 1% in the following Decimal Square, you would shade 1 square. Shade 110% in the Decimal Square.
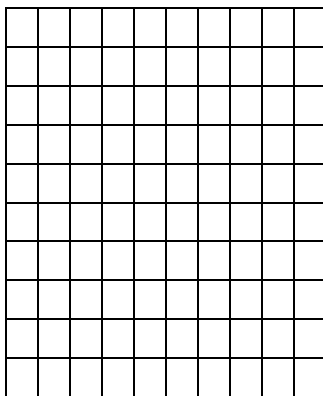
Section 3:   Changing Percents to Decimals

We use the meaning of percent to change a percent to a decimal number.

Example 3 :   Change 37% to a decimal.

Solution: 37% means 37 "per hundred" or 37 over 100.

$$37\% \; = \; \frac{37}{100}$$ *We used the meaning of % to convert to a fraction with*
denominator 100.

$$= \; 0.37$$ *We divided 37 by 100 to obtain the decimal.*

◆

Example 4 :   Change 8.75% to a decimal.

Solution: 8.75% means 8.75 "per hundred" or 8.75 over 100.

$$8.75\% \; = \; \frac{8.75}{100}$$ *We used the meaning of % to convert to a fraction with*
denominator 100.

$$= \; \frac{875}{10000}$$ *Multiplied numerator and denominator by 100 to*
eliminate of the decimal in the numerator.

$$= \; 0.0875$$ *Divided 875 by 10000 to obtain the decimal.*

◆

From the last two examples, you can see that the % symbol has been dropped and the decimal point has been moved two places to the **left**. Since % means "per hundred," you will always move the decimal two places to the left when changing a percent to a decimal.

> **To change a percent to a decimal, drop the % symbol and move the decimal point two places to the left.**

Activity 3

Change the following percents to decimals.

a)   98%   _____   d)   0.9%   _____

b)   1%   _____   e)   150%   _____

c)   10.07%   _____

Section 4:   Changing Decimals to Percents

To change percents to decimals we moved the decimal point two places to the left; to change decimals to percents we have to do the opposite.   For example, 12% written as a decimal is 0.12, which means to change 12% back to a decimal we move the decimal point two places to the right and insert the % symbol:

$$0.12 \ = \ 12\%$$

To change a decimal to a percent, we move the decimal point two places to the right and insert the % symbol.

Example 5 :   Change the following decimals to percents.

a) 0.45   b) 0.005   c) 5

Solution: a) $0.45 \ = \ 45\%$

b) $0.005 \ = \ 0.5\%$

c) $5 \ = \ 5.00 \ = \ 500\%$ ♦

Class Activity 4

Change the following decimals to percents.

a) . 67   _____   b)   0.8   _____

c)   0.0007   _____   d)   12   _____

Section 5: Changing Percents to Fractions

Since we know that percent means "per hundred," to change a percent to a fraction we just **drop** the % symbol and write the original number **over 100**.

Example 6 :   Change 40% to a fraction.

Solution: $40\% = \frac{40}{100}$ *Percent means "per hundred."*

$= \frac{2}{5}$ *Write the fraction in lowest terms.*   ♦

Example 7 :   Change 10.5% to a fraction.

Solution: $10.5\% = \frac{10.5}{100}$ *Percent means "per hundred."*

$= \frac{10.5 \text{ x } 10}{100 \text{ x } 10}$ *We want a whole number in the numerator, so we*
find an equivalent fraction by multiplying the
numerator and denominator by 10.

$= \frac{105}{1000}$

$= \frac{21}{200}$ *Write the fraction in lowest terms.*

♦

Example 8 :   Change $8\frac{3}{4}$ % to a fraction.

Solution: $8\frac{3}{4}$ % $= \frac{8\frac{3}{4}}{100}$ *Percent means "per hundred."*

$= \frac{\frac{35}{4}}{100}$ *Change $8\frac{3}{4}$ to an improper fraction.*

$= \frac{35}{4} \div 100$ $\frac{a}{b}$ *means a ÷ b.*

$= \frac{35}{4} \text{ x } \frac{1}{100}$ *Dividing by 100 is the same as multiplying by $\frac{1}{100}$.*

$= \frac{35}{400}$ *Multiply numerators, then denominators.*

$= \frac{7}{80}$ *Write the fraction in lowest terms.*   ♦

Activity 5

Change the following percents to fractions. Show all work **neatly** and place your answer in the space provided.

a) 65%   a) _____

b) 100%   b) _____

c) 240%   c) _____

d) 34.6%   d) _____

e) $7\frac{1}{3}$ %   e) _____

f) $\frac{1}{2}$ %   f) _____

Section 6:   Changing Fractions to Percents

We can change a fraction to a decimal either by first changing the fraction to a decimal and then change the decimal to a percent or we can use a proportion. The following examples illustrate both methods.

Example 9 :   Change $\frac{9}{10}$ to a percent.

Solution: **Method 1.** Let's first change $\frac{9}{10}$ to a decimal and then change the decimal to a percent.

We change $\frac{9}{10}$ to a decimal by dividing 9 by 10.

$$
\begin{array}{r}
0.9 \\
10\overline{)9.0} \\
9\,0 \\
\hline
0
\end{array}
$$

We now have to change 0.9 to a percent by moving the decimal point two places to the right and inserting the % symbol.

Hence, $\frac{9}{10} = 0.9 = 90\%$

Method 2. From the definition of percent, we know that percent is a fraction with a denominator of 100, $x\% = \frac{x}{100}$. Thus to express a fraction as a percent we must find an equivalent fraction that has a denominator of 100.

$$\frac{9}{10} = \frac{x}{100}$$

$10x = 9(100)$ _Cross-multiply_

$10x = 900$

$x = \frac{900}{10} = 90$ _Divide both sides of the equation by 10._

Hence, $\frac{9}{10} = \frac{90}{100} = 90\%$.   _Substitute 90 in for x in the original equation._

♦

Example 10 :   Change $\frac{5}{8}$ to a percent.

Solution: **Method 1.**   **Divide numerator by denominator** $(5 \div 8).$

$$
\begin{array}{r}
0.625 \\
8\overline{)5.000} \\
\underline{48} \\
20 \\
\underline{16} \\
40 \\
\underline{40} \\
0
\end{array}
$$

Hence, $\frac{5}{8} = 0.625 = 62.5\%$

Method 2.   **Set up a proportion and solve.**

$\frac{5}{8} = \frac{x}{100}$

$8x = 500$

$x = \frac{500}{8} = 62.5$ or $62\frac{1}{2}$.   (*Divide 500 by 8.*)

Hence, $\frac{5}{8} = \frac{62\frac{1}{2}}{100} = 62\frac{1}{2}$ %

Activity 6

Change the following fractions to percents. Show all work **neatly** and place your answer in the space provided.

a) $\frac{3}{4}$

      Method 1   Method 2

b) $\frac{17}{40}$

      Method 1   Method 2

c) $\frac{5}{12}$

      Method 1   Method 2

Section 7: Percent Problems

The general form of a percent problem is $\boxed{\textbf{% of total is part}}$ or $\boxed{\dfrac{\%}{100} = \dfrac{part}{total} = \dfrac{is}{of}}$.

We can solve percent problems by using 2 different methods.

Method 1:   Use a proportion.

Method 2:   Translate into a mathematical equation.

We will use these 2 methods to solve the following percent problems.

$\boxed{\textbf{Example 11}}$ **:**   15% of 40 is what number?

Solution: Here we know the percent (15%) and the total (40) and we are looking for the part.

Method 1: $\boxed{\dfrac{\%}{100} = \dfrac{part}{total} = \dfrac{is}{of}}$

$\dfrac{15}{100} = \dfrac{n}{40}$ *Place 15 over 100 and drop the % symbol.*

Substitute 40 in for the total.

$100n = 15(40)$ *Cross multiply.*

$\dfrac{100n}{100} = \dfrac{600}{100}$ *Divide both sides by 100.*

$n = 6$

Therefore, 15% of 40 is $\boxed{6}$ **.**

Method 2:   Using the general form, $\boxed{\textbf{% of total is part}}$,
we translate the following to math symbols.

% $\rightarrow$ change to a decimal

of $\rightarrow$ change to multiplication

is $\rightarrow$ change to an equal sign

Now that we have a mathematical equation,
we solve for the unknown.

To answer the given question, 15% of 40 is what number?

$$\downarrow \quad \downarrow \quad \downarrow \downarrow \qquad \downarrow$$

We translate to get: $\qquad 0.15 \quad \cdot \quad 40 \ = \ $ n

Then we solve for n by multiplying: $\quad 6 = $ n

Once again, 15% of 40 is $\boxed{6}$. $\qquad \qquad \blacklozenge$

$\boxed{\textbf{Example 12}}$: $\qquad$ 25% of what number is 50.2?

Solution: $\qquad$ Here we know the percent (25%) and the part (50.2) and we are looking for the **total**.

Method 1: $\qquad \boxed{\dfrac{\%}{100} = \dfrac{\textbf{part}}{\textbf{total}} = \dfrac{\textbf{is}}{\textbf{of}}}$

$$\frac{25}{100} = \frac{50.2}{n}$$

Place 25 over 100 and drop the % symbol. Substitute 50.2 for the part.

$$25n = (50.2)(100)$$

Cross multiply.

$$\frac{25n}{25} = \frac{5020}{25}$$

Divide both sides by 25.

$$n = 200.8$$

Therefore, 25% of $\boxed{\textbf{200.8}}$ is 50.2.

Method 2: $\qquad \boxed{\textbf{\% of total is part}}$

To answer the given question, 25% of what number is 50.2?

$$\downarrow \quad \downarrow \quad \downarrow \qquad \downarrow \ \downarrow$$

We translate to get: $\qquad 0.25 \quad \cdot \qquad n \qquad = \quad 50.2$

To solve for n, we must divide both sides of the equation by 0.25.

$$\frac{0.25n}{0.25} = \frac{50.2}{0.25}$$

$$n = 200.8$$

Once again, 25% of $\boxed{\textbf{200.8}}$ is 50.2. $\qquad \qquad \blacklozenge$

Example 13 : 45 successful free throws is what % of 68 attempts? (Round the answer to the nearest tenth of a percent.)

Solution: Here we know the **part** (45 successful free throws) and the **total** (68 attempts) and we are looking for the **percent**.

Method 1: $\dfrac{\%}{100} = \dfrac{\text{part}}{\text{total}} = \dfrac{\text{is}}{\text{of}}$

$$\frac{n}{100} = \frac{45}{68}$$
$$68n = 45(100)$$
$$\frac{68n}{68} = \frac{4500}{68}$$
$$n = 66.2$$

Therefore, 45 successful free throws is approximately **66.2%** of 68 attempts.

Method 2: **% of total is part**

What % of 68 is 45?	*Write a question for the given problem.*
$n \cdot 68 = 45$	*Translate the question to an equation.*
$\dfrac{n \cdot 68}{68} = \dfrac{45}{68}$	*Divide both sides by 68.*
$n \approx 0.662 \approx 66.2\%$	*Write the decimal as a percent.*

Once again, 45 successful free throws is approximately **66.2%** of 68 attempts. ◆

Activity 7

___

Answer the following questions, using either method.

1. 40% of 170 is what number?

2. What number is $5\frac{1}{4}\%$ of 2500?

3. 29% of what number is 34? Round the answer to the nearest tenth.

___

4.   32 is 50% of what number?

5.   What percent of 200 is 60?

6.   If there are 346 students in the 8th grade at Keene Middle School and 198 are girls, what percent of the 8th grade is girls? Round the answer to the nearest tenth of a percent.

Section 8:   Percent Application Problems

These problems are different from the ones in the last section in that you are not given the question explicitly. Instead, you need to write a question from the given word problem. Once you determine the question being asked, you use the same procedure as in section 7.

Example 14 :   About 65% of the students at a certain college receive some form of financial aid. How many of the 9280 students enrolled this year are receiving aid? (*Source: Prealgebra by Lial and Hestwood)*

Solution: Using the general form, **% of total is part** , we can write the question:

65% of 9280 total students is what number receiving aid?

Method 1:   **Method 2:**

$$\frac{65}{100} = \frac{n}{9280}$$ $$0.65 \cdot 9280 = n$$

$$\frac{100n}{100} = \frac{603200}{100}$$ $$6032 = n$$

$$n = 6032$$

Therefore, **6032 students** are receiving aid.   ◆

Example 15 : A frozen dinner advertises that only 18% of its calories are from fat. If the dinner contains 55 calories from fat, what is the total number of calories in the dinner? Round the answer to the nearest whole number of calories. (*Source: Prealgebra by Lial and Hestwood)*

Solution: Using the general form, $\boxed{\text{% of total is part}}$, we can write the question:

18% of what total calories is 55 calories from fat?

Method 1:

$$\frac{18}{100} = \frac{55}{n}$$

$$18n = 55(100)$$

$$\frac{18n}{18} = \frac{5500}{18}$$

$$n = 306$$

Method 2:

$$.18 \cdot n = 55$$

$$\frac{.18n}{.18} = \frac{55}{.18}$$

$$n = 306$$

Therefore, there are approximately $\boxed{\textbf{306 total calories}}$ in the frozen dinner. ◆

Activity 8
_____

Answer the following questions: (*Source: Prealgebra by Lial and Hestwood)*

1. There are 50 points on the first math test. Hue's score was 83% correct. How many points did Hue earn?

2. The rainfall in the Red River Valley was 33 inches this year. The average rainfall is 30 inches. This year's rainfall is what percent of the average rainfall?

3. A newspaper article stated that 648 pints of blood were donated at the blood bank last month, which was only 72% of the number of pints needed. How many pints of blood were needed?

_____

Exercise for Percents

Do all the exercises on separate paper, showing all work neatly.

1.   Write the following as an equivalent fraction without the % symbol.

   a)   9%   b)   1.7%   c)   470%

2.   Write the following using the % symbol.

   a) $\dfrac{3}{100}$ b $\dfrac{50\frac{1}{3}}{100}$

3.   For each of the following Decimal Squares, determine how much is shaded.
      Write your answer as a percent, decimal, and a fraction.
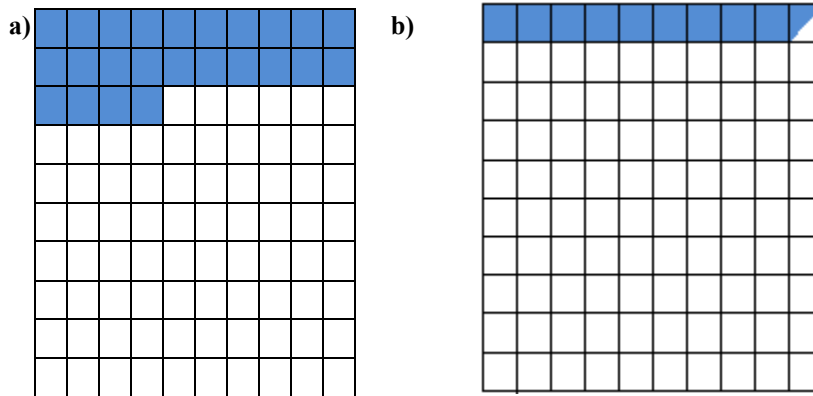
4.   Change the following percents to decimals.

   a) 91%   b) 8%   c) 0.11%   d) 104%

5.   Change the following decimals to percents.

   a)   .85   b)   0.2   c)   0.0014   d)   7

6.   Change the following percents to fractions.

   a) 76%   b) 67.3%   c) $3\frac{3}{4}$ %

7.   Change the following fractions to percents.

   a) $\dfrac{7}{80}$ b) $\dfrac{9}{17}$

8.   What number is 20% of 180?

9. What percent of 16 is 9?

10. 16 is 20% of what number?

11. 24 is what percent of 16?

12. 79% of 980 is what number?

13. How much HCL (hydrochloric acid) is in a 60-millimeter bottle that is marked 80% HCL?

14. Complete the following chart with the correct common fraction, decimal, or percent equivalents.

	Common Fraction	Decimal	Percent
a)	_____	.5	_____
b)	1/3	_____	_____
c)	_____	_____	25%
d)	3/4	_____	_____
e)	_____	.2	_____
f)	_____	_____	40%
g)	4/5	_____	_____
h)	_____	.16666	_____
i)	_____	_____	$83\frac{1}{3}$ %
j)	1/8	_____	_____
k)	_____	.375	_____
l)	_____	_____	87.5%

15. There are about 55,000 words in Webster's Dictionary, but most educated people can identify only 20,000 of these words. What percent of the words in the dictionary can these people identify? (*Source: Prealgebra by Lial and Hestwood*)

16.   A survey at an intersection found that of 2200 drivers, 38% were wearing seat belts. How many drivers in the survey were wearing seat belts? (*Source: Prealgebra by Lial and Hestwood*)

17.   So far this season, Kevin Garnett has made 76% of his free throws. He has made 133 free throws. How many has he attempted? (*Source: Prealgebra by Lial and Hestwood*)

<u>PERCENTS</u>

<u>Activity 1:</u>

a)   5% = $\dfrac{5}{100}$

b)   7.6% = $\dfrac{7.6}{100}$

c)   270% = $\dfrac{270}{100}$

d) $\dfrac{16}{100}$ = 16%

e) $\dfrac{7\frac{1}{5}}{100}$ = $7\frac{1}{5}$%

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<u>Activity 2:</u>

a)   Percent   4%

b)   Percent $21\frac{1}{2}$% = 21.5%

   Decimal   0.04

   Decimal   0.215

   Fraction $\dfrac{4}{100} = \dfrac{1}{25}$

   Fraction $\dfrac{21.5}{100} = \dfrac{215}{1000} = \dfrac{43}{200}$

c)   Percent   12.75%

   Decimal   0.1275

   Fraction $\dfrac{51}{400}$

Shade the indicated percent. Write the percent as a decimal and as a fraction.

a)   shade $15\frac{1}{2}$ squares

b)   shade $6\frac{1}{4}$ squares

   decimal   0.155

   decimal   0.0625

   fraction $\dfrac{155}{1000} = \dfrac{31}{200}$

   fraction $\dfrac{625}{10000} = \dfrac{1}{16}$

To shade 110%, you would need to shade the entire Decimal Square plus one column of another Decimal Square.

Activity 3:

a)   98% = 0.98   d)   0.9% = 0.009

b)   1% = 0.01   e)   150% = 1.50

c)   10.07%= 0.1007

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Activity 4:

a)   0 .67 = 67%   b)   0.8 = 80%

c)   0.0007 = .07%   d)   12 = 1200%

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Activity 5:

a) $65\% = \dfrac{65}{100} = \dfrac{13}{20}$

b) $100\% = 1$

c) $240\% = \dfrac{240}{100} = 2\dfrac{40}{100} = 2\dfrac{2}{5}$

d) $34.6\% = \dfrac{34.6}{100} = \dfrac{346}{1000} = \dfrac{173}{500}$

e) $7\dfrac{1}{3}\% = \dfrac{7\frac{1}{3}}{100} = 7\dfrac{1}{3} \div 100 = \dfrac{22}{3} \bullet \dfrac{1}{100} = \dfrac{22}{300} = \dfrac{11}{150}$

f) $\dfrac{1}{2}\% = \dfrac{\frac{1}{2}}{100} = \dfrac{1}{2} \div 100 = \dfrac{1}{2} \bullet \dfrac{1}{100} = \dfrac{1}{200}$

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Activity 6:

a)   *Method 1*   *Method 2*

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

.75 = 75%

$\dfrac{3}{4} = \dfrac{x}{100}$

$3(100) = 4x$

$300 = 4x$

$x = \dfrac{300}{4} = 75$

$\dfrac{75}{100} = 75\%$

Witkowski and Phillips

b)   *Method 1*   *Method 2*

$$\begin{array}{r} 0.425 \\ 40\overline{)17.00} \\ \underline{160} \\ 100 \\ \underline{80} \\ 200 \\ \underline{200} \\ 0 \end{array}$$

$$\frac{17}{40} = \frac{x}{100}$$

$$17(100) = 40x$$

$$1700 = 40x$$

$$x = \frac{1700}{40} = 42.5$$

$$\frac{42.5}{100} = 42.5\%$$

$$0.425 = 42.5\%$$

c)   *Method 1*   *Method 2*

$$\begin{array}{r} .416 \\ 12\overline{)5.000} \\ \underline{48} \\ 20 \\ \underline{12} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

$$\frac{5}{12} = \frac{x}{100}$$

$$5(100) = 12x$$

$$500 = 12x$$

$$x = \frac{500}{12} = 41.\overline{6}$$

$$\frac{41.\overline{6}}{100} = 41.\overline{6}\% \ or \ 41\frac{2}{3}\%$$

$$0.416 = 41.\overline{6}\% \ or \ 41\frac{2}{3}\%$$

****

Activity 7:   (The proportion method will be used to solve these problems.)

1.
$$\frac{40}{100} = \frac{x}{170}$$
$$100x = (40)(170)$$
$$100x = 6800$$
$$x = \frac{6800}{100} = 68$$

2.
$$\frac{5.25}{100} = \frac{x}{2500}$$
$$100x = (5.25)(2500)$$
$$100x = 13125$$
$$x = \frac{13125}{100} = 131.25$$

3.
$$\frac{29}{100} = \frac{34}{x}$$
$$29x = (34)(100)$$
$$29x = 3400$$
$$x = \frac{3400}{29} = 117.2$$

Witkowski and Phillips

$$4.\quad \frac{50}{100} = \frac{32}{x}$$
$$50x = (32)(100)$$
$$50x = 3200$$
$$x = \frac{3200}{50} = 64$$

$$5.\quad \frac{x}{100} = \frac{60}{200}$$
$$200x = (60)(100)$$
$$200x = 6000$$
$$x = \frac{6000}{200} = 30$$
$$so,\ \frac{30}{100} = 30\%$$

$$6.\quad \frac{x}{100} = \frac{198}{346}$$
$$346x = (198)(100)$$
$$346x = 19800$$
$$x = \frac{19800}{346} = 57.2$$
$$so,\ \frac{57.2}{100} = 57.2\%$$

****

<u>Activity 8:</u>   (The translation method will be used to solve these problems.)

1.   83% of 50 is what number?

$$0.83 \bullet 50 = x$$
$$41.5 = x$$

Therefore, Hue earned 41.5 or $41\frac{1}{2}$ points.

2.   33 is what percent of 30?

$$33 = x \bullet 30$$
$$\frac{33}{30} = x$$
$$1.1 = x$$

To answer this question, we must write 1.1 as a percent. 1.1 = 110%

Therefore, this year's rainfall is 110% of the average rainfall.

3.   72% of what number is 648?

$$0.72 \bullet x = 648$$
$$x = \frac{648}{0.72} = 900$$

Therefore, there were 900 pints of blood needed.