OGAP Multiplicative Framework

(Draft June 2012)

Depending upon the strength of multiplicative reasoning students may move back and forth between using multiplicative, transitional, additive, and non-multiplicative strategies as they interact with different problem structures and problem situations. (Kouba & Franklin, 1995; VMP OGAP, 2006)

PROBLEM CONTEXTS

Equal groups
Equal measures
Equations
Measure conversions
Multiplicative change
Multiplicative comparison
Patterns
Properties
Rate
Rectangular area
Volume

PROBLEM STRUCTURES

Types of Items
Application/context
Concept/property

Multiplicative Representations
Equal groups
Arrays
Area
Open area
Linear

Number of Factors
Two factors
More than two factors

Complexity of Numbers

Factors
Single digit
Multiple digit
Powers of ten
Fractions/decimals

Divisors
Single digit
Multiple digit
Powers of ten
Fractions/decimals

Understanding and Use of Properties
Associative
Commutative
Distributive
Equality
Identity
Multiplicative Inverse

Understanding and Use of Relationships
Addition-Multiplication
Doubling and Halving
Model - Equation
Model - Model
Multiples and Factors
Meanings of remainders

Types of Division
Partitive
Quotative

Language
Natural (e.g., every)
Mathematical (e.g., per)

Problem Situations

Refers to known and unknown information in a problem.
For example, in equal group problems there might be an unknown product, or an unknown number of equal groups, or an unknown quantity in a group.
See page 4 for some examples. Also see the CCSSM page 89.

ABOUT OGAP FRAMEWORKS

OGAP Frameworks are based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems. OGAP references are found at http://www.margepetit.com/petit_pdfs/OGAPReferences3.pdf.

There are three major elements to an OGAP Framework that should be considered when analyzing student work or making instructional decisions:
1) problem contexts
2) problem structures
3) evidence in student work

This page identifies problem situations and problem structures for multiplication and division problems. Pages 2 and 3 are tools to help teachers classify evidence in student work, including classroom discussions, and make instructional decisions. Page 4 has samples of different problem contexts and situations.

For students to become strong multiplicative reasoners they must interact with a range of problem situations and problem structures. The CCSSM specifically identifies problem contexts at targeted grades on a progression from equal groups and measures, and area situations at grades 2 and 3 to measurement conversions, volume, and multiplicative change and patterns, and multiplicative comparison problem situations at grades 4 and 5. This progression, among other things, is designed to prepare students to engage in proportional situations using multiplicative reasoning.

Consistent with the CCSSM the OGAP Frameworks on pages 2 and 3 show a progression from the link between repeated addition and multiplication in an equal groups model to the development of efficient and generalizable multiplicative strategies through the open area model, and understanding of place value, properties of operations, and relationships.

As students interact with new concepts, new problem situations, new structures, and more complex problem solving situations they may move back and forth between multiplicative, transitional, additive, and non-multiplicative strategies. This is important evidence to use for instructional decision making. For example, a student may consistently solve equal group problems using a Multiplicative Strategy regardless of the complexity of the numbers, but you may find that the same student adds factors (Non-Multiplicative Strategy) when solving multiplicative change problems.

This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project (OGAP) which was funded by NSF (EHR-0227057) and the US DOE (S366A020002). *Version 18 June 2012*

OGAP Multiplicative Reasoning Framework - Multiplication (Draft June 2012)

Multiplicative Strategies

Algorithms

Partial Products

16
X 42
12
20
240
400
672

Traditional

^{2} ^{1}
16
X 42
32
640
672

Distributive Property

16 x 4 = 4 (10 + 6) = 4(10) + 4(6) = 40 + 24 = 64

Associative Property

(8 x 2) x 5 = 8 (2 x 5) = 8 x 10 = 80

Commutative Property

16 x 4 = 4 x 16

Doubling and Halving

16 x 4 = 8 x 8 = 64

Known or Derived Fact

6 x4 = 24

Powers of Ten

5 x 400 = 5 x 4 x 10 x 10

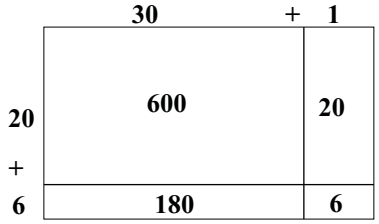
Transitional Multiplicative Strategies

Transitional

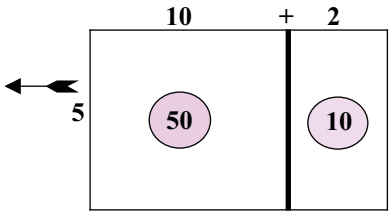
Open Area Model

Considers both dimensions of the array and area model.

26 x 31 = 806

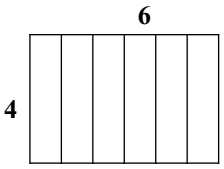


12 x 5 = 60



Area Model

6 x 4 = 24



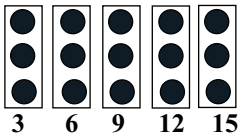
Considers BOTH dimensions of an area model moving away from needing to see every square unit.

Early Transitional

Skip Counting

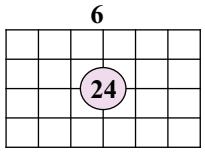
3, 6, 9, 12, 15

Equal groups in an array



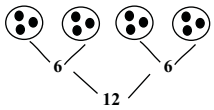
Only considers one dimension of an array or area model.

Area Model 6 x 4 = 24

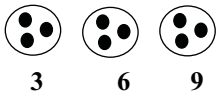


Considers BOTH dimensions of an array or area model.

Building up



3 + 3 + 3 + 3
6 + 6
12



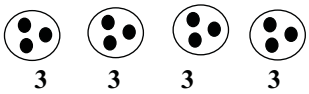
Additive Strategies

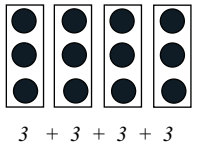
Additive

Repeated addition with or without a model

3 + 3 + 3 + 3 = 12

3 x 4 = 12

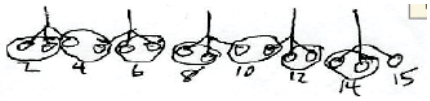




Subitizing in small groups

Modeling - Counting by ones

Modeling - Counting by sub groups 3 x 5 = 15



Unitizes into groups and sub-groups

Applies Understanding of Place Value, Properties, and Relationships

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Non-Multiplicative Strategies

Adds or subtracts factors
Models factors incorrectly
Uses incorrect operation
Not enough information
Guesses

Underlying Issues/Errors

Misinterprets the remainders
Does not consider the reasonableness of solution
Units inconsistent or missing
Calculation error
Place value error
Vocabulary error
Property or relationship error
Equation error

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OGAP Multiplicative Reasoning Framework - Division (Draft June 2012)

Multiplicative Strategies

Algorithms

Partial Quotients

```
17 | 585
   | 340   x 20
   | 245
   | 170   x 10
   |  75
   |  68   x 4
   |   7   34 r 7
```

```
          4
         10
         20  34 r 7
17 | 585
   | 340
   | 245
   | 170
   |  75
   |  68
   |   7
```

Traditional

```
     34 r 7
17 | 585
   | 51↓
   |  75
   |  68
   |   7
```

Distributive Property

35 ÷ 7 = (21 + 14) ÷ 7 = 3 + 2 = 5

Treats the remainder appropriately given problem situation

Inverse relationship between multiplication and division

35 ÷ 7 = 5 **7 x ? = 35**

Known or Derived Fact

21 ÷ 7 = 3

Transitional Multiplicative Strategies

Transitional

Inefficient partial quotients

```
61 | 756   12 R 24
   | 183   x3
   | 573
   | 183   x3
   | 390
   | 183   x3
   | 207   x3
   | 183
   |  24
```

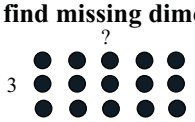
Trial and error to find a quotient

```
61 | 756
   | 732
   |  24
     12 R 24
```

2 x 61 = 122
20 x 61 = 1220
4 x 61 = 244
8 x 61 = 488
15 x 61 = 915
10 x 61 = 610
12 x 61 = 732

Early Transitional

Models in an array to find missing dimensions

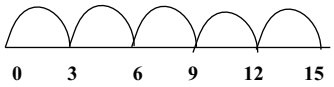


15 ÷ 3 = ? **3 x ? = 15**

Skip Counts to find the number of "skips" with and without a model

15 ÷ 3 = 5

3, 6, 9, 12, 15 (5 skips)



Additive Strategies

Additive

Repeated subtraction or adding up to

```
61 | 350   5 r 45
   |  61
   | 289
   |  61
   | 228
   |  61
   | 167
   |  61
   | 106
   |  61
   |  45
```

```
 61
+61
122
+61
183
+61
244
+61
305
+61
366
```

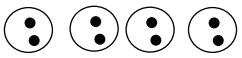
```
350
-305
 r 45
```

Sharing out in equal groups as repeated subtracted or addition

There are 8 cookies to share equally with 4 children. How many cookies does each child get?

8 cookies ÷ 4 children =2 cookies per child

Represents the 4 children with circles and then fills them equally

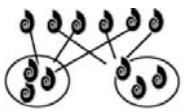


Twenty-four cookies were put into bags of 4 cookies each. How many bags were filled?

24 cookies ÷ 4 cookies per bag = 6 bags

Pulls out 4 cookies at a time until 24 cookies are used.

Sharing out by ones



Sharing out randomly by subsets

Sharing 4, then 2, then 5 and so on

Early

Applies Understanding of Place Values, Properties, and Relationships

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OGAP Sample Problem Contexts and Situations (Draft June 2012)

Important:
The sample problem contexts and situations do not include the full range of each problem situation or context.

Multiplication Examples	Division Examples How many in each group?(partitive) How many groups? (quotative)
Equal group, measurement conversion, equal measure, and rate problems involve applying a rate. number of groups/measurements/quantities x rate = total number Multiplicative change, multiplicative patterns and multiplicative comparison involve a multiplicative scale factor. original x scale factor/multiplier = result Area and volume problems involve using dimensions in either an area or volume situation. *Problem situations are identified with an * and are in italics in the examples below.*	
Equal Group *****Unknown product (total number of crayons)* Mark bought 12 boxes of crayons. Each box contained 8 crayons. How many crayons were there all together? 12 boxes x 8 crayons per box = ? crayons	* Unknown number in each group (crayons per box)* Mark had a box of 64 crayons. He shared the crayons equally with 4 people. How many crayons did each person get? (partitive) 64 crayons = 4 people x ? crayons per box
Equal Measures * Unknown product(total length in inches)* It takes 14 inches of ribbon to make one bow. How many inches of ribbon will it take to make 7 bows? 7 bows x 14 inches per bow = ? inches	* Unknown number of groups (number of designs)* Sam ha s 15 yards of material. He is making a design that needs 3 yards per design. How many designs can Sam make? (quotative) 15 yards ÷ 3 yards per design = ? designs
Measurement Conversion *****Unknown product (length in inches)* Tammy is 5 feet tall. How many inches tall is Tammy? 5 feet x 12 inches/foot = ? inches	
Rates *****Unknown product (total dollars)* Sam works at the grocery store. He is paid $7.00 per hour. He worked 22 hours last week. How much money did Sam earn last week? 22 hours x $7.00/hour = ? dollars	* Unknown rate (dollars per hour)* Sam earned $154.00 last week. He worked 22 hours. How much did Sam earn per hour? (partitive) $154.00 = 22 hours x ? dollars per hour
Multiplicative Comparison * Larger unknown (height in inches)* The students in Mrs. Gilbert's class planted bean and corn seeds. The bean plants grow 3 times faster than the corn plants. When the corn plants measure 2 inches, how tall will the bean plants be? 2 inches x 3 = ? height of bean plants	* Multiplier unknown (scale factor unknown)* Bill's garden is 240 square feet. Leslie's garden is 20 square feet. How many times greater is Bill's garden than Leslie's garden? (quotative) 240 square feet ÷ 20 square feet = ? times bigger
Multiplicative Change/Patterns * Larger unknown (length)* A 5-inch piece of elastic is stretched 3 times its length. How long is the elastic after it is stretched? (4 times, 5 times, *n* times) 5 inches x 3 = ? (total length)	* Smaller unknown (original length)* A piece of elastic stretches to 3 times its length. When fully stretched it is 57 inches long. What is its original length? (partitive) Total length (57 inches) ÷ 3 = ? (original length)
Area ***** Unknown product (area in square feet)* Linda's kitchen floor measures 12 feet by 7 feet. How many tiles (1 square foot) are needed to cover the floor? 12 feet x 7 feet = ? (total area in square feet)	* Unknown factor (length of kitchen floor)* Linda's kitchen floor is 150 square feet. The length of one dimension is 10 feet. What is the length of the other dimension of the kitchen floor? 150 square feet ÷ 10 feet = ? (length of other dimension in feet)
Volume * Unknown volume (volume of water the tank can hold* The dimensions of a fish tank are 10 inches x 12 inches by 18 inches. If you filled the tank to the top, how much water would be in the tank? 10 inches x 12 inches x 18 inches = ? amount of water in tank	* Unknown factor (height of container)* A rectangular container holds 250 cubic inches of liquid. the base of the container is 10 inches and the depth 5 inches. What is the height of the container? 10 inches x 5 inches x ? inches = 250 cubic inches
OGAP Equation Example: **6 x 5 = 30** Write a story problem that goes with this equation. OR match a story problem to an equation.	**OGAP Property Example:** Ann knows the answer to 9 x 5. Explain how can she use this information to solve 45 ÷ 9?

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