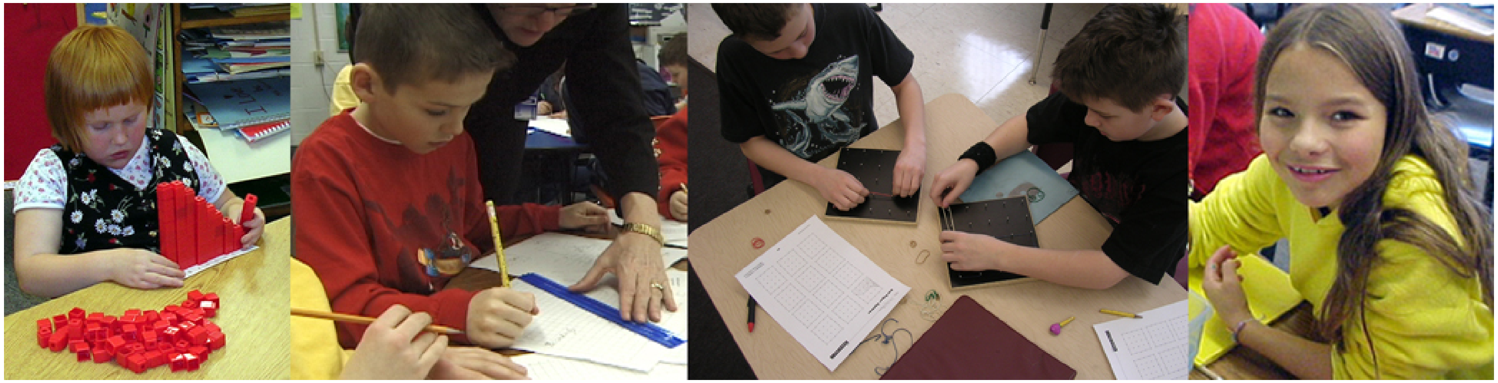

# Learning Progressions and the CCSSM: Current Thinking and Examples (OGAP and CBAL)

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THE VERMONT INSTITUTES
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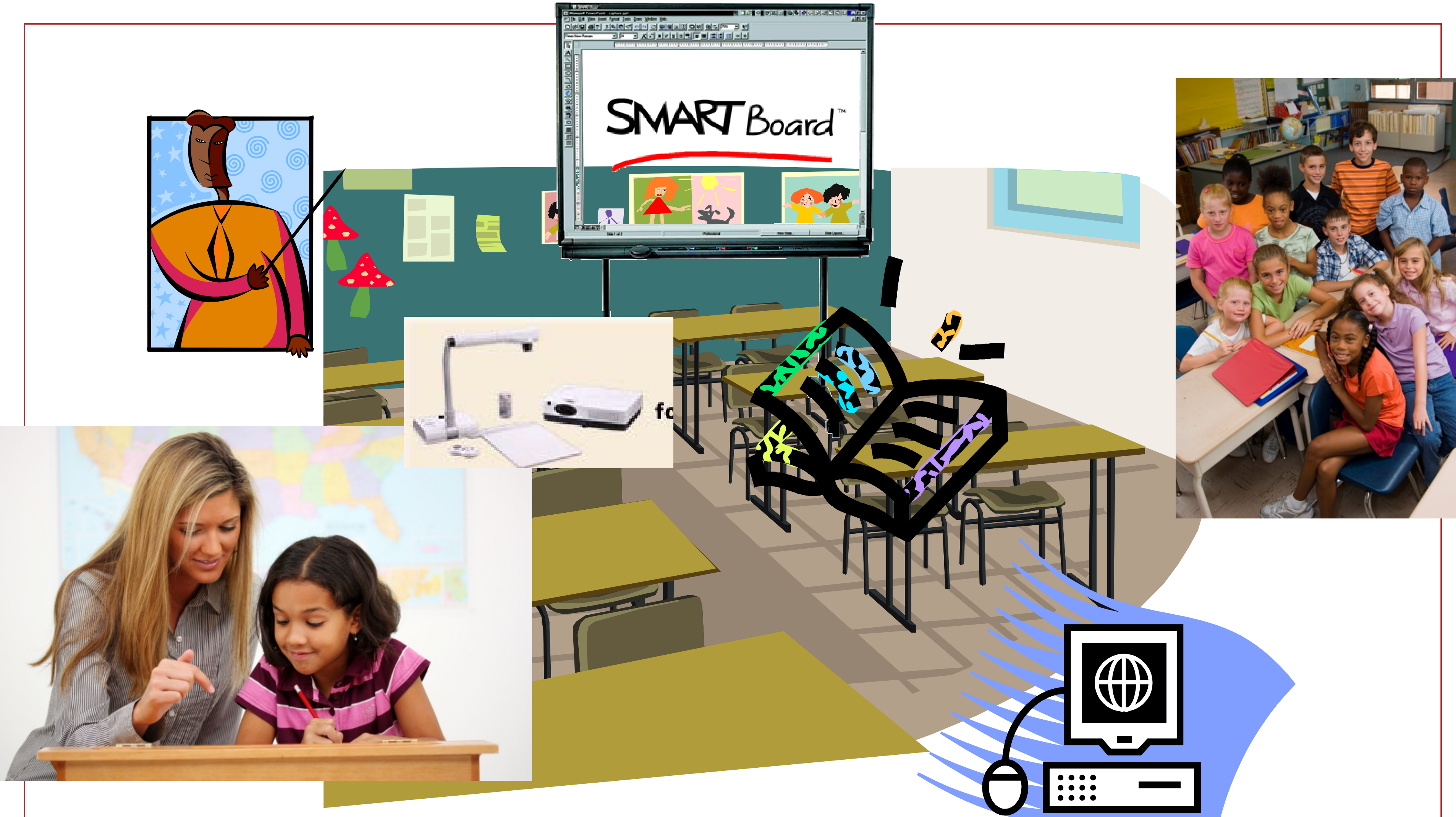
MPC
Educational Consulting

OGAP was developed as a part of the Vermont Mathematics Partnership funded by The NSF (Award Number EHR-0227057) and the US DOE (S366A0200002))

***Vermont Mathematics Partnership***




In the end – it is  the evidence of student thinking not just from assessment questions, but also from classroom discussions and activities that informs instructional decision making.

# Take Aways!



- **Teacher knowledge** about the research/learning trajectories is fundamental – this involves a real commitment to PD, NOT just creating tools and materials, but substantive professional development.

- **Evidence of Student Thinking** - it is the evidence of student thinking not just from assessment questions, but from classroom discussions and activities that informs instructional decision making.

- **Formative assessment** is a powerful tool when it is implemented systematically and intentionally coupled with the above.

- **Learning progressions** are valuable only if they are truly accessible to teachers and students

- The **CCSS** and OGAP Framework

# In 1.5 hours…



## **What can be done** ….

- …provide participants with the big idea of learning progressions and some examples (OGAP and CBAL)

## **What <u>cannot</u> be done**…

- … provide participants with a deep understanding of the details and potential implications of learning progressions and OGAP
- …be sure that participants understand the difference between formative and summative assessment.

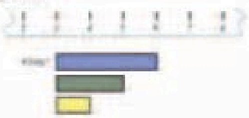
# Learning Progression/Trajectory



“

- “..descriptions of children’s thinking and learning in a specific mathematical domain, and a related conjectured route through a set of instructional tasks designed to move children through a developmental progression of thinking, created with the intent of supporting children’s achievement of specific goals in that mathematical domain.” (Clements & Sarama, 2004)

Illustration of a portion of a learning trajectory describing the growth of children's understanding of linear measurement

| Trajectory Level | Conceptual Structures | Instructional Tasks | Example of Instructional Task |
|---|---|---|---|
| Age 6: **End-to-End** Length Measurer (**EE**): Lays units end-to-end. May not recognize the need for equal-length units. Needs a complete set of units to span a long object. | Expects that lengths can be composed as repetitions of shorter lengths. This initially only applies to small numbers of units. The scheme is enhanced by the growing conception of length measuring as sweeping through large units coordinated with composing a length with parts (unit sticks). | 1) Provide incomplete sets of linear objects to span the length of an object (see next column) 2) Use relatively large objects as units and build a ruler with pen length units 3) Compare two objects indirectly using only shorter objects. 4) Provide the student with a contiguous set of yellow strips taped in a row to find length for comparisons. 5) Draw a ruler and mark it with ticks and numerals to match units (in or cm). | Item 1 from prior column: How long is the blue strip, compared to one of the yellow strips? Can you find out without moving any more yellow strips?  |
| Age 7: **Length Unit Relater and Repeater (URR)**: Measures by repeated use of a unit (*initially may be imprecise as with broken ruler tasks*). Relates size and number of units explicitly, but may use units of varying lengths. Can add lengths to obtain the length of a whole. Iterates a single unit to measure. Uses rulers with minimal guidance. | Action schemes include the ability to iterate a mental unit along an object. Cardinal values are connected to space units for small quantities but weaker beyond these. With the support of a perceptual context, scheme can predict that fewer larger units will be required. | 1) Given a drawing of a 5-unit segment, ask students to draw a 3-unit length line segment (Cannon, 1992). 2) Have students create units of units, such as a "footstrip". 3) Repeat measures using different-sized units and relate them. 4) Broken ruler task. 5) Measure with a covered ruler section to prevent unit counts. 6) Compare wire around tile perimeter with tile edge as units. 7) Ask students who are counting points instead of intervals (over by one each time) to draw and measure decreasing sequences of segments. [next column illustrates this] | Item 7 from prior column: If the blue strip is reported to be 4 units long by a struggling student, have them find the length of the green and yellow strips. If the student reports 3 and 2 for these measures, ask them to draw a 1 unit long segment. Or, ask them how many 2 unit yellow strips would make up a 3 unit green strip. This should prompt them to re-measure and build up the yellow as 1 unit, the green as 2 units, and the blue as 3 units.  |
| Age 8: **Consistent Length Measurer (CLM)**: Finds length on a bent path as the sum of its parts. Measures consistently, knowing need for identical units, partitions of unit, zero point on rulers, and accumulation of distance. May coordinate units and subunits. | Scheme includes the ability simultaneously to imagine an object's length as a total extent and a composition of units. Only allows equal-length units. Can measure from starting points other than zero on a ruler. Units themselves can be partitioned to increase precision. | 1) Use a physical unit and a ruler to measure line segments and objects that require both an iteration and subdivision of the unit. 2) Build sub-units to fourths and eighths. 3) Discuss how to deal with leftover space, to count it as a whole unit or as part of a unit when units do not iterate to an integer value for length. [next column illustrates an integration of all 3 points] | Draw 4 different paths that are shorter than 5 and one half inch and longer than 5 and one quarter inch. Put the paths in order, and describe the length of each one in inches. |


*Sources:*

Barrett, J., Clements, D., Sarama, J., Cullen, C., McCool, J., Witkowski, C., & Klanderman, D. (in press). Evaluating and Improving a Learning Trajectory for Linear Measurement in Elementary Grades 2 and 3: A Longitudinal Study. *Mathematical Thinking and Learning.*

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children.* New York: Routledge.

RPC(M):45

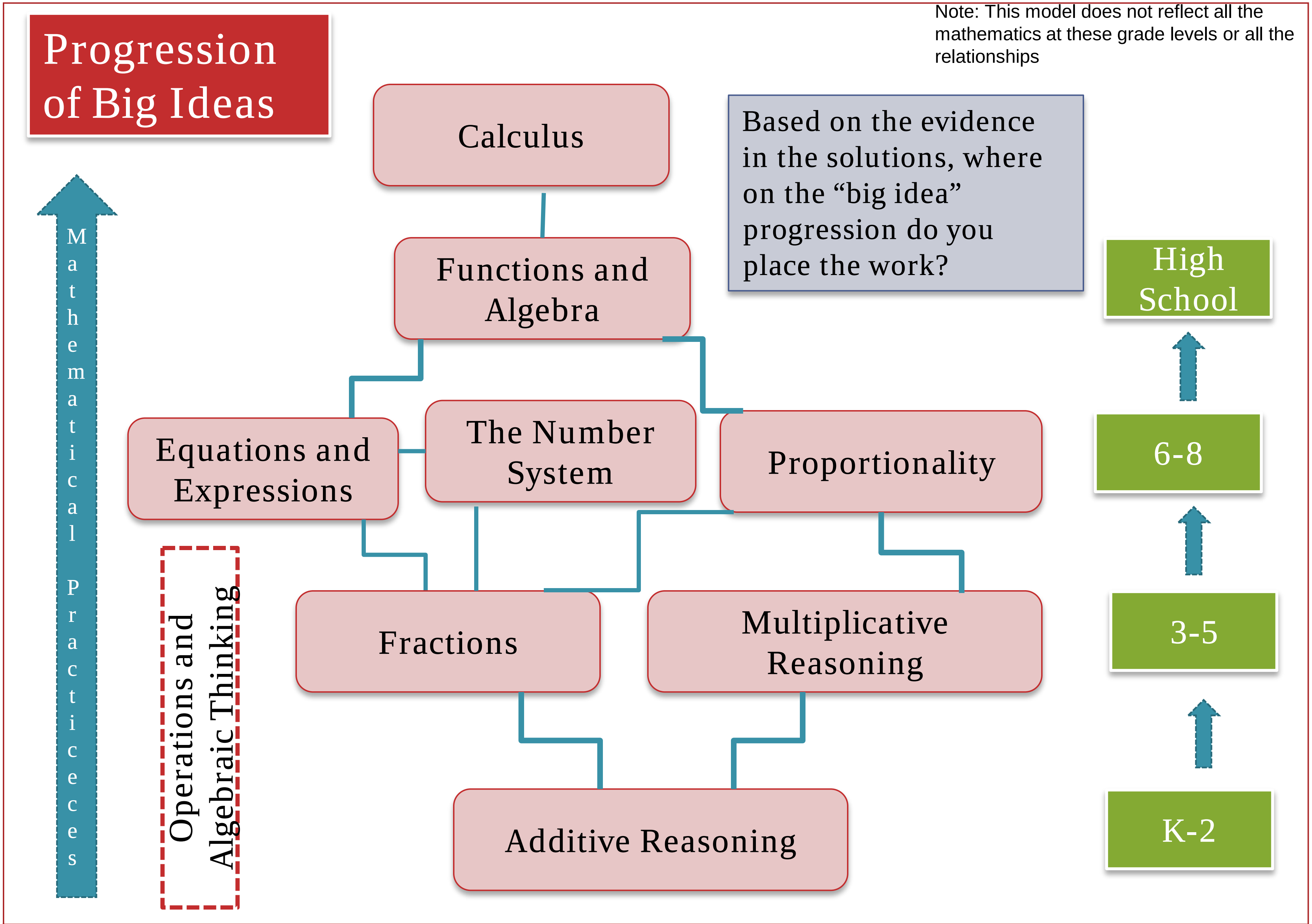
Jamie is in charge of purchasing pancake mix for the Club's Annual Breakfast fundraiser. He is using the ratio table below to determine the amount of mix to purchase:

| Number of pancakes | 12 | 24 | 36 | 120 | 400 |
|---|---|---|---|---|---|
| Cups of Pancake Mix | $1\frac{3}{4}$ | $3\frac{1}{2}$ | $5\frac{1}{4}$ | $17\frac{1}{2}$ | |
| Milk | $1\frac{1}{4}$ | $2\frac{1}{2}$ | $3\frac{3}{4}$ | $12\frac{1}{2}$ | |

The Club expects to make about 400 pancakes. How many cups of mix does Jamie need?

Explain your reasoning.

From OGAP item bank developed as a part of the Vermont Mathematics Partnership funded by The NSF (Award Number EHR-0227057) and the US DOE (S366A0200002))


Progression of Big Ideas
Note: This model does not reflect all the mathematics at these grade levels or all the relationships
Calculus
Based on the evidence in the solutions, where on the "big idea" progression do you place the work?
Functions and Algebra
High School
Equations and Expressions
The Number System
Proportionality
6-8
Mathematical Practices
Operations and Algebraic Thinking
Fractions
Multiplicative Reasoning
3-5
Additive Reasoning
K-2

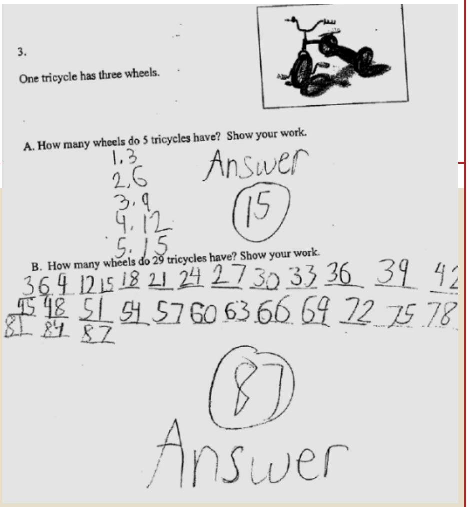
# Learning Progression – **The promise**



- "If teachers have a fine-grained understanding of the steps of learning and the problems students are likely to encounter, then they have a basis for identifying evidence in student work to locate where a learner is in the progression and for responding to this evidence in ways calculated to help them deal with any problem and keep moving ahead." (G. Postlewait, CCSSO.)

# Student Work

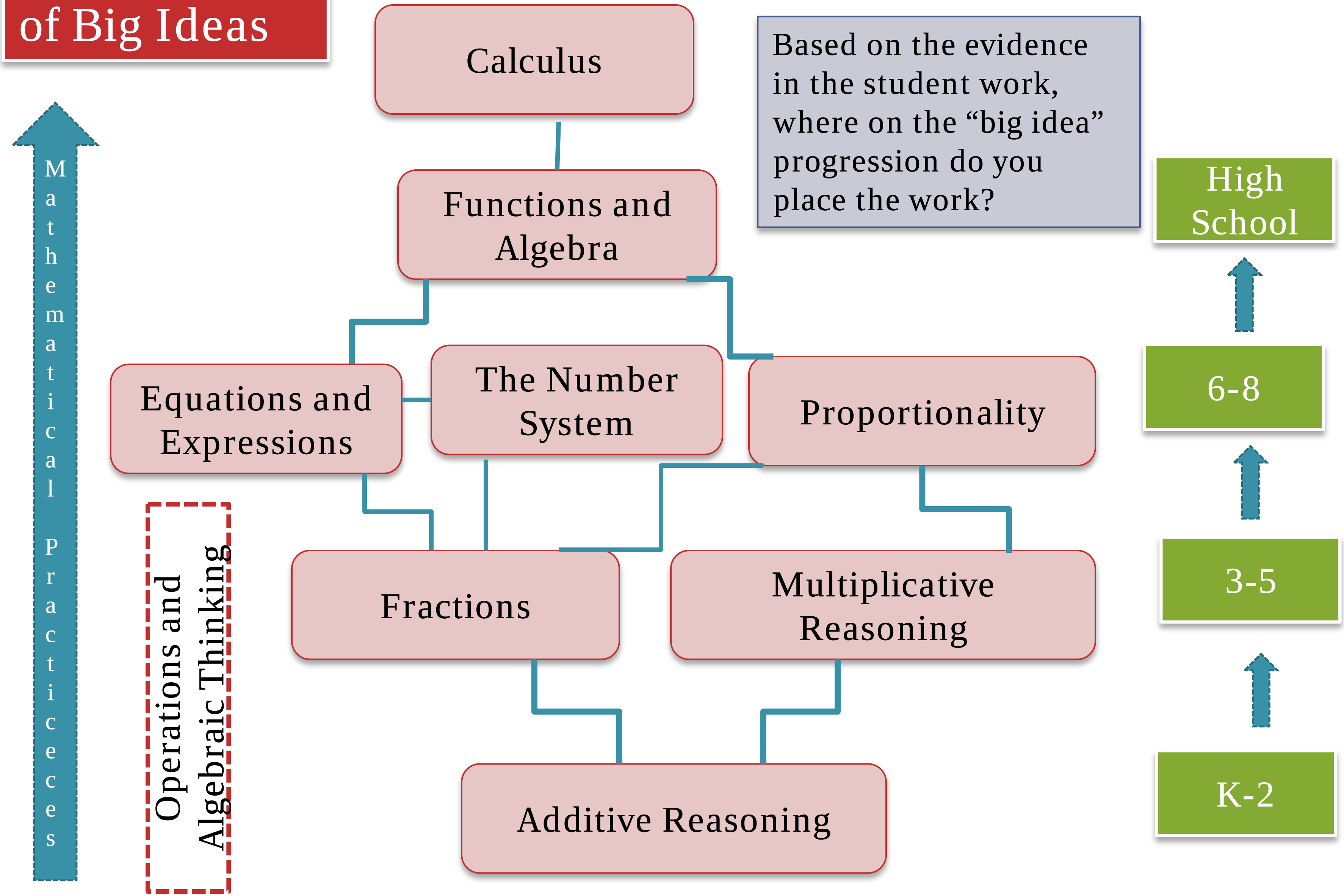





- With a partner review the set of student work. As you review the work collect observations about the responses.
  - Do all students use the same strategy as they solve the problem?
  - Do all students use the same strategy as they solve the 2 different problems?
  - Are there some strategies that are more efficient than others?
  - Other observations?

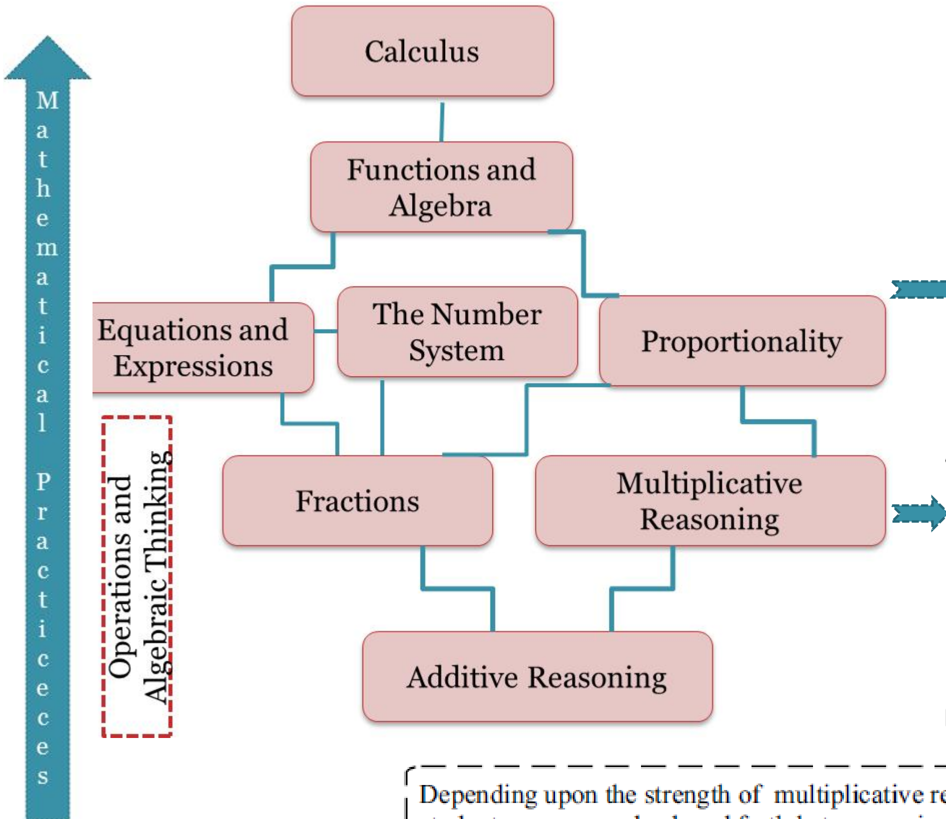
# Progression of Big Ideas

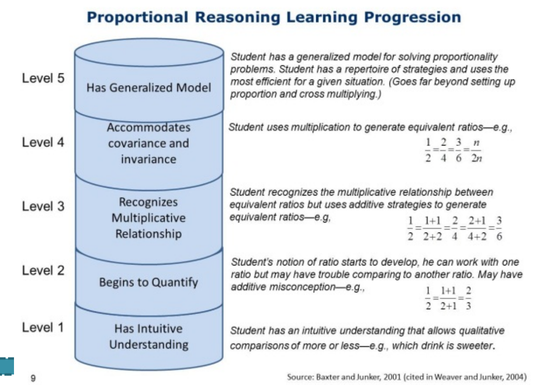
Note: This model does not reflect all the mathematics at these grade levels or all the relationships



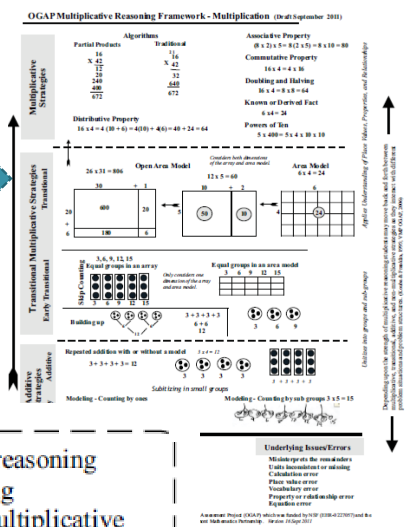
# Learning Progressions

"Are careful, empirically based descriptions of the ways in which students' conceptions and skills actually grow." (Postlewait, CCSSO Presentation)




Depending upon the strength of proportional reasoning students may move back and forth between using proportional, transitional, and non-proportional strategies as they interact with different problem structures and problem situations. (Cramer, Post & Currier, 1993; Karplus, Pulos & Stage, 1983; VMP OGAP, 2006 & 2007)

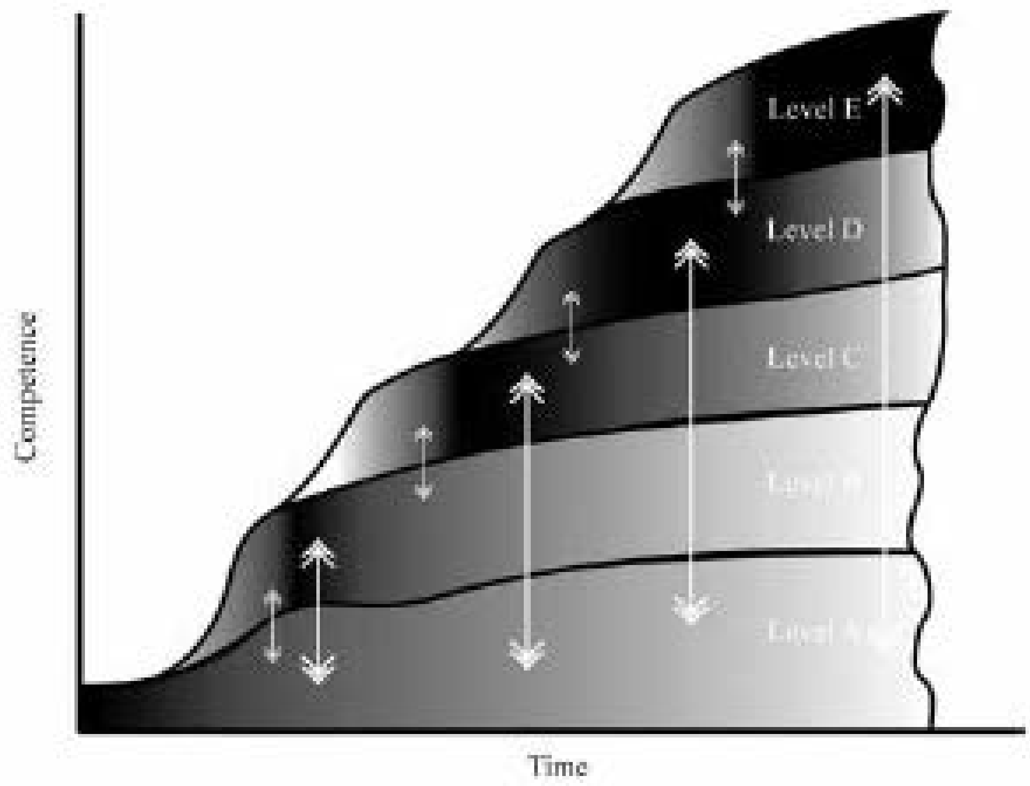


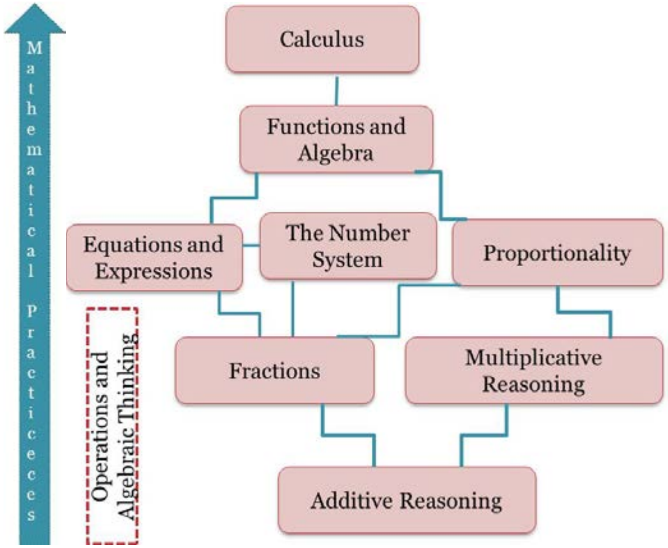

Depending upon the strength of multiplicative reasoning students may move back and forth between using multiplicative, transitional, additive, and non-multiplicative strategies as they interact with different problem structures and problem situations. (Kouba & Franklin, 1995; VMP OGAP, 2006)

**Implications:** Progressions are not clean nor are they unidirectional. They involve the interaction of development of mathematical concepts within and across domains, cognitive complexity of problems, and the structures in the problem.

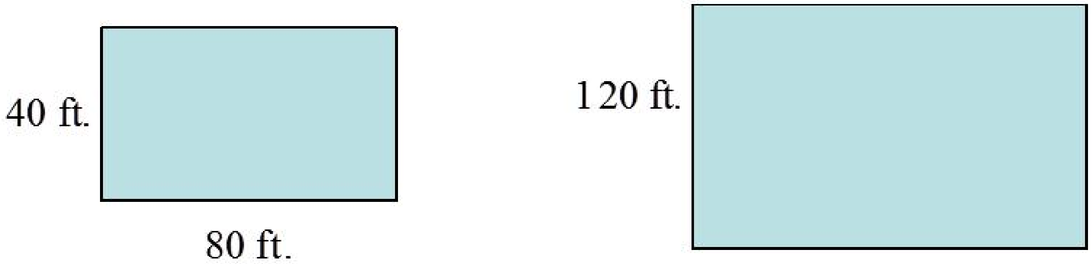
Mathematical Practices

Illustration of the theoretical account of developing competence over time, perhaps as short a timespan as 2 years, or as long as 10 years:





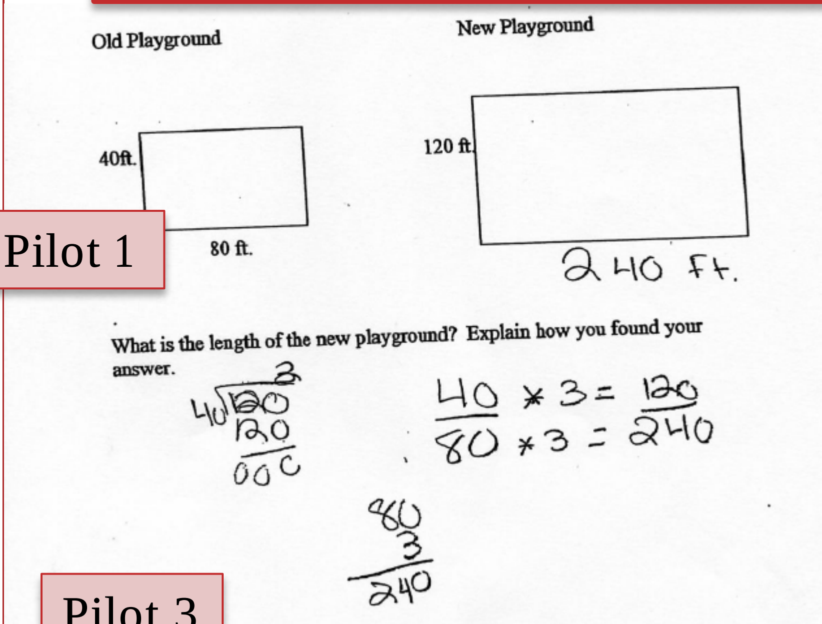

PILOT 1: A school is enlarging its playground. The dimensions of the new playground are proportional to the dimensions of the old playground



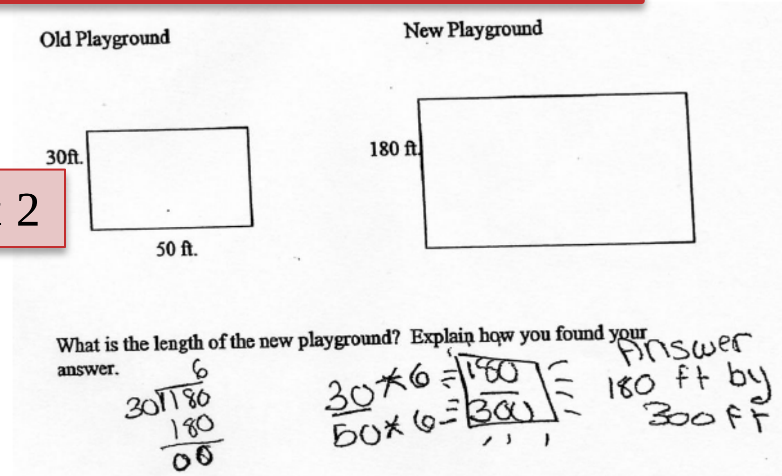

What is the length of the new playground?

A school is enlarging its playground. The dimensions of the new playground are proportional to the dimensions of the old playground

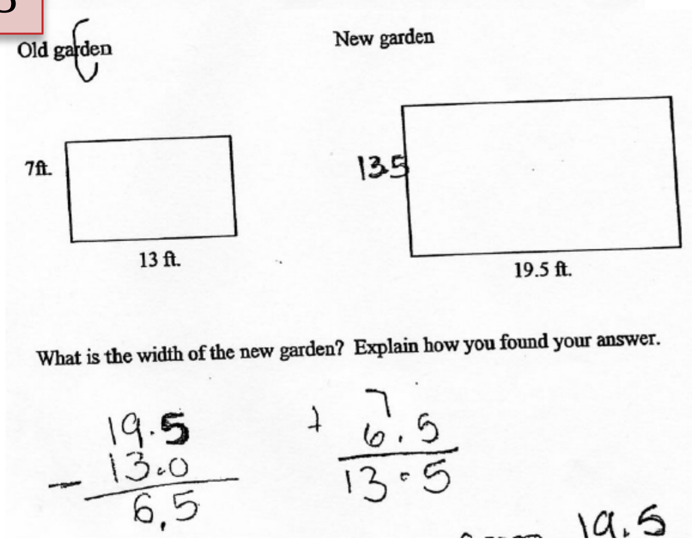
Pilot 1




Pilot 2




Pilot 3




## OGAP 2006 Pilot 7th grade (*n*=153)

| | Multiplicative Relationships "within" and "between" Ratios | Percent of Correct Responses |
|---|---|---|
| **Pilot 1** | Both integral | 80% |
| **Pilot 2** | One integral, one non-integral | 65% |
| **Pilot 3** | Both non-integral | 35.5% |

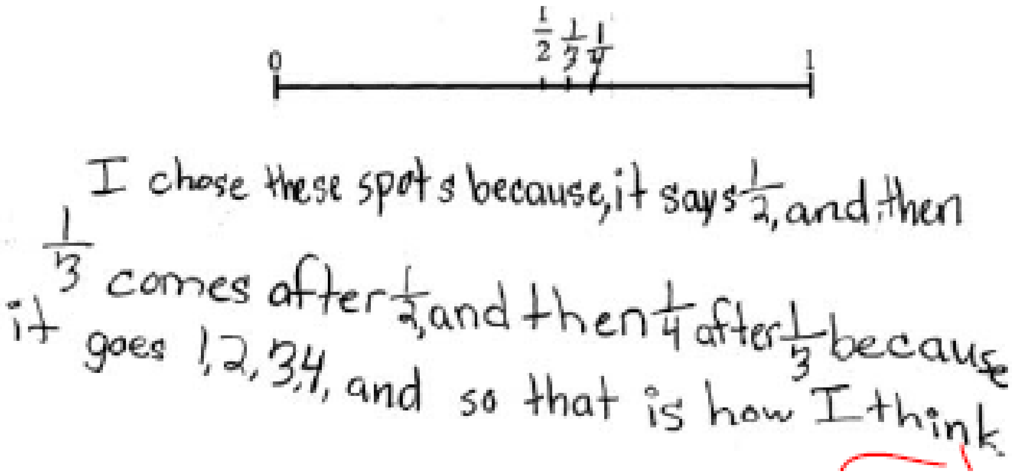
# To Succeed.. (Daro et al, 2011)



There needs to be careful attention & timely interventions by a well-trained teacher who understands how children learn mathematics, where they struggle & what to do about it.

- To adapt, a teacher must know how to get students to reveal where they are in terms of what they understand & what their problems might be.
- A teacher must have specific ideas of how students are likely to progress, including what prerequisite knowledge & skill they should have mastered, & how they might be expected to go off track or have problems.
- A teacher would need to have, or develop, ideas about what to do to respond helpfully to the particular evidence of progress & problems they observe.

*According to research, some students may see a fraction as two whole numbers (e.g., ¾ as a 3 and 4) inappropriately using whole number reasoning, not reasoning with a fraction as a single quantity.* (Behr, M., Post, T., Lesh, R., and Silver, E. (1983); Behr, Wachsmuth and Post, (1984); *VMP OGAP Study (2005))*




A) The sum of $\frac{1}{12} + \frac{7}{8}$ is closest to:

a) 20

b) 8

c) $\frac{1}{2}$

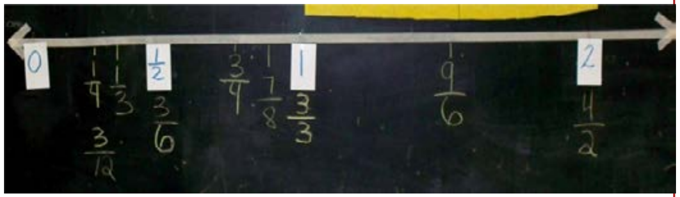
d) 1

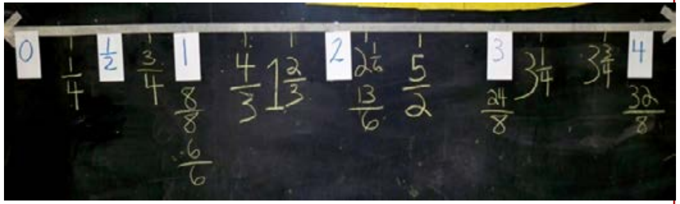
Use words, pictures, or diagrams to explain your answer.

$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$ is closest to 20.

# Examples of teacher interventions (response to inappropriate whole number reasoning)

- Use modeling to build concepts
- Emphasis on number line
- Emphasis on relative magnitude of fractions using modeling and other reasoning strategies





**OGAP Whole Number Reasoning Sub-study(2005)**

| | Percentage of Students | Average number of incorrect responses |
|---|---|---|
| Pre-assessment | 85% (33/39) | 4.1 (33 students) |
| Post assessment | 18% (7/39) | 1.8 (7 students) |

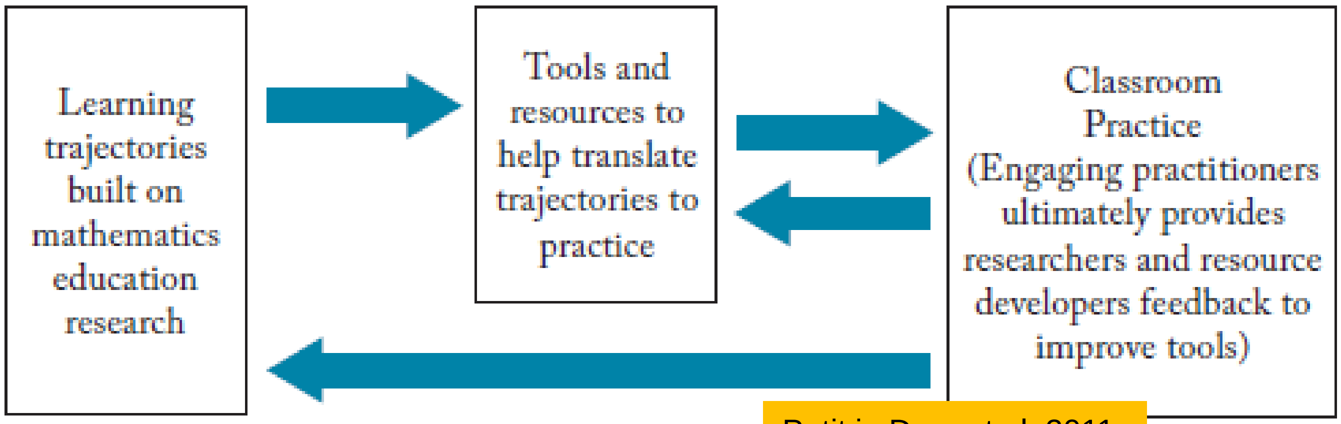
OGAP Exploratory Studies (2004, 2005) and 2006-2008 Roll-outs

# To utilize progressions teachers ….



- Need knowledge of mathematics education research/progressions
- Tools and resources to help translate trajectories/progressions into practice.

Figure 1. Transfer of Knowledge from Learning Trajectory Research into Classroom Practice




Petit in Daro et al, 2011

# Teachers are ready!

1990-2000s

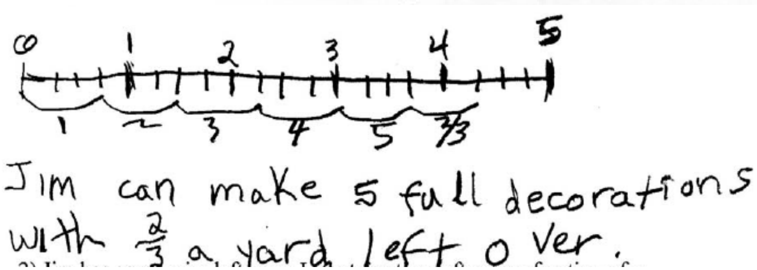
Wow – look at all the ways students can solve the problem

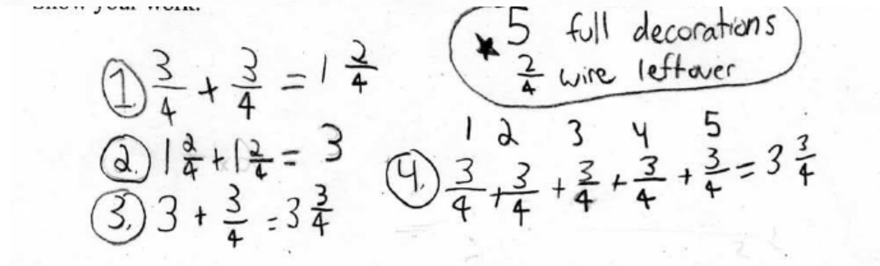
Jim is making decorations for a school dance.

He has $4\frac{1}{4}$ yards of wire. Each decoration needs $\frac{3}{4}$ of a yard of wire.

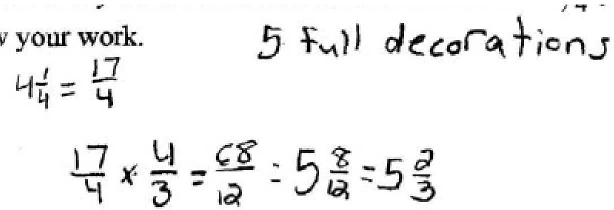
1) How many full decorations can Jim make from $4\frac{1}{4}$ yards of wire?

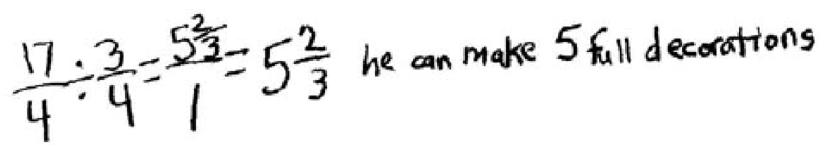
Show your work.




Show your work.




What do these different solutions mean? What do I do?

NOW

# Teachers say understanding the math education research help them... (OGAP 2005 Study)



- Understand the purposes of activities in math programs;
- Understand evidence in student work used to inform instruction;
- Strengthen and focus first wave instruction;
- Respond to evidence in student work as instruction proceeds.

# Two Examples and an ILN Opportunity



- Vermont Mathematics Partnership Ongoing Assessment Project (OGAP)
- Cognitively Based Assessment of, for, and as Learning (CBAL)

# OGAP (2003 – Present)



- Studies (2004, 2005, 2006, 2007 involving 100s of teachers, thousands of students) Analysis of studies is continuing)

- Currently aspects of OGAP being used in Vermont, Alabama, Nebraska, Michigan, Ohio, and Amman, Jordan.

# OGAP is a systematic and intentional formative assessment system in mathematics based on math education research.



- Gathering information about pre-existing knowledge through the use of a **pre-assessment**;
- **Analyzing pre-assessment** to guide unit planning; and
- **A continuous and intentional system** of instructing, probing with instructionally embedded questions, analysis, and instructional modification.

**Grades 2 - 8**

- **Fractions**
- **Multiplicative reasoning**
- **Proportionality**

# In place and in use for all 3 mathematical topics



- Pre-assessments and ongoing questions
- Tools and strategies to analyze student work including Frameworks/progressions
- Professional development workshop materials and resources to communicate research and support the use of OGAP formative assessment system

# OGAP was Developed Based on Four Principles

***Principle # 1:*** *Build on pre-existing knowledge* (*How People Learn (2000)* National Research Council)

***Principle # 2****: Learn (and assess) for Understanding*

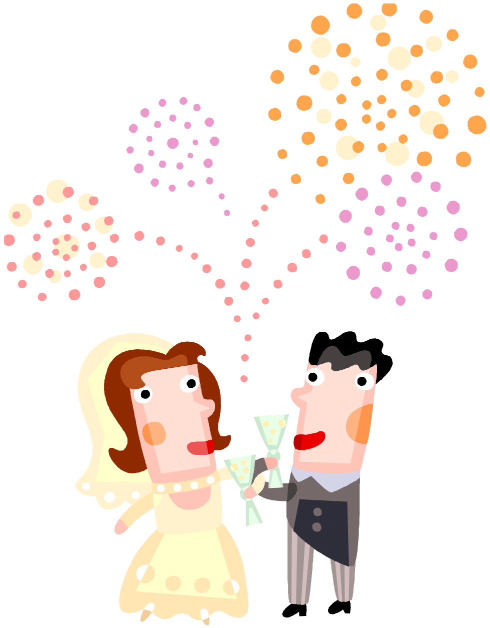
(Adding it Up! (*2001) National Research Council*)

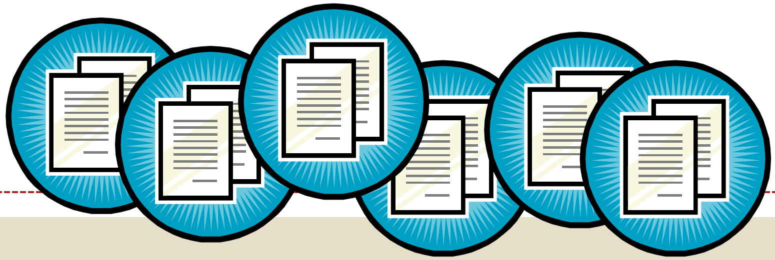
***Principle # 3:*** *Use Frequent Formative Assessment*
**(***Inside the Black Box, (2001) Black, P, and Wiliam, D.)*

***Principle # 4:*** *Build Assessment on Mathematics Education Research (Knowing What Students Know (2001) National Research Council)*

It is not formative assessment alone OR knowledge of mathematics education research alone…

**…but the marriage of the two that empowers teachers**







**Hundreds of research articles distilled into a frameworks/learning progressions**

## In design of materials

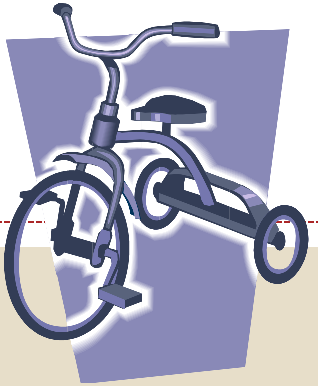
- formative assessment items
- professional development materials (case studies, activities, essays)
- Book and articles
- Frameworks/learning progressions

## In work with educators

- analyze student work
- inform instructional decisions
- help understand the purposes of activities in mathematics programs

# The tricycles



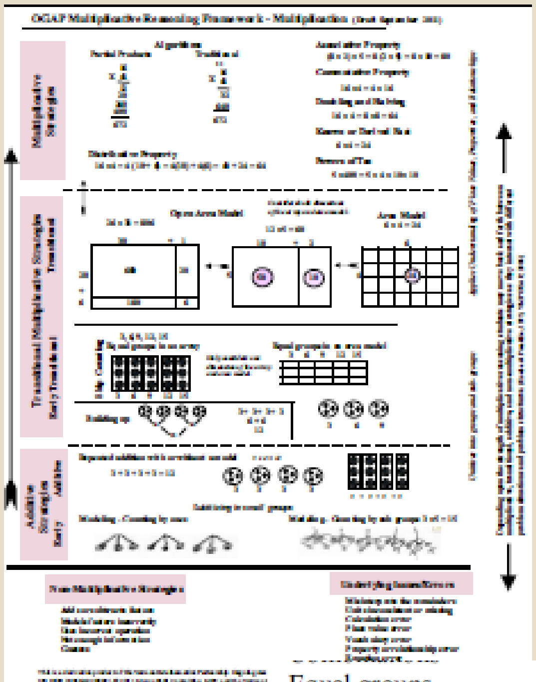


A) How many wheels do 5 tricycles have?

B) How many wheels do 29 tricycles have?

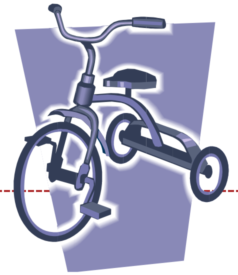
- What is the problem situation?
- What are other problem structures to consider?
- What are strategies that were used? Where are they on the OGAP Framework?

# CCSS Whole Number Multiplication Link to the OGAP Framework

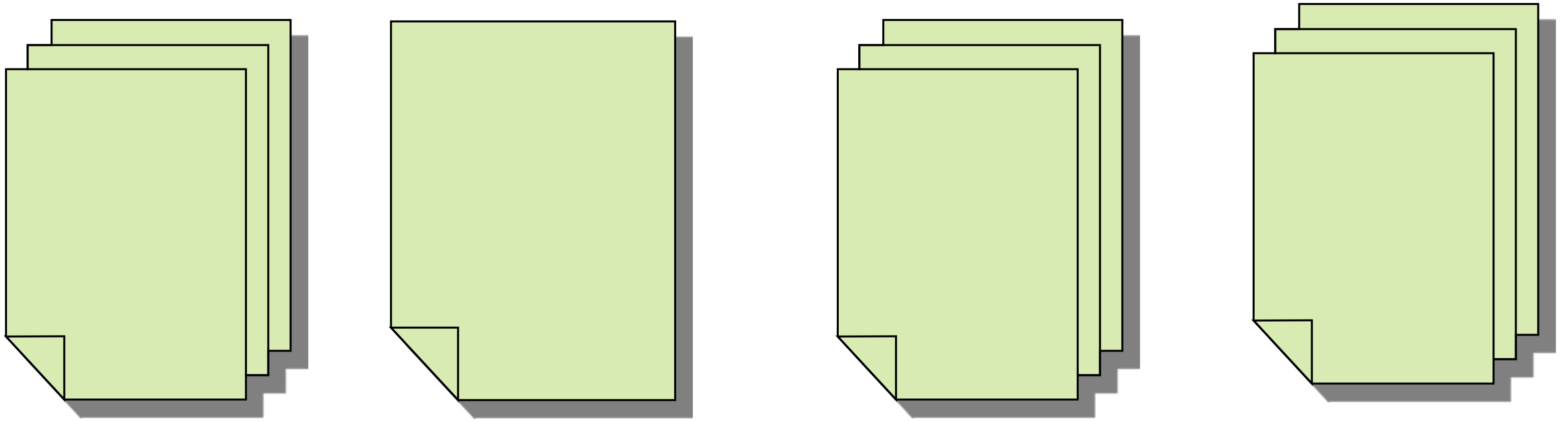


Equal groups
Equal measures
Equations
Measure conversions
Multiplicative change
Multiplicative comparison
Patterns
Properties
Rate
Rectangular area
Volume

| | Problem Situations | Strategies |
|---|---|---|
| Grade 2 | Equal groups | Repeated addition with an array |
| Grade 3 (factors within 100) | Equal groups, arrays, equal measures, beginning area | Properties of operations, drawings, equations |
| Grade 4 (1 digit x 4 digit, and 2 digit x 2 digit) | Multiplicative comparison, measurement conversion within systems, area | Place value and properties… using equations, rectangular arrays, and/or area model, equations |
| Grade 5 (fluently) | Scaling (multiplicative change), area, volume, patterns, conversions between systems | Standard algorithm, equations |



# Analyzing student work – the OGAP Sort



While OGAP has item banks of short focused questions for fractions, multiplicative reasoning, and proportionality that teachers can use for formative assessment probes (e.g., exit or entry cards) ...

... the OGAP frameworks/learning progressions can be used to analyze evidence of in student thinking/work during day to day instruction, and problems of any complexity (in a given math topic).

# OGAP is a systematic and intentional formative assessment system in mathematics.



- Gathering information about pre-existing knowledge through the use of a **pre-assessment**;
- **Analyzing pre-assessment** to guide unit planning; and
- **A continuous and intentional system** of instructing, probing with instructionally embedded questions, analysis, and instructional modification.

**Grades 2 - 8**

- **Fractions**
- **Multiplicative reasoning**
- **Proportionality**

# CBAL Mathematics Overview

- Work began at middle school grade levels
- Now expanding into elementary and high school grade levels
- Online scenario-based assessment tasks (both summative and formative)
- Based on mathematical competency models
- Augmented by learning progressions

# CBAL Mathematics and the Common Core State Standards

- CBAL's content competencies span multiple middle school grade levels in the CCSS (somewhat greater overlap with CCSS 7, 8)
- Consistency between CBAL's cross-cutting mathematical processes and CCSS standards of mathematical practice
- CBAL learning progressions are consistent with several critical areas of focus in CCSS at grades 7-8

# CBAL - Learning Progressions (Provisional)

Middle School Grade Levels

- Linear Functions*
- Proportionality
- Equality and Variable
- Statistics (various models still under development)

- Elementary School Grade Levels
  - Fractions and Decimals (under development)

- High School
  - Quadratic Functions* (under development)

*Based on general functions model

## CBAL

### Proportional Reasoning Learning Progression

| Level | Stage | Description |
|---|---|---|
| Level 5 | Has Generalized Model | *Student has a generalized model for solving proportionality problems. Student has a repertoire of strategies and uses the most efficient for a given situation. (Goes far beyond setting up proportion and cross multiplying.)* |
| Level 4 | Accommodates covariance and invariance | *Student uses multiplication to generate equivalent ratios—e.g.,* $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{n}{2n}$ |
| Level 3 | Recognizes Multiplicative Relationship | *Student recognizes the multiplicative relationship between equivalent ratios but uses additive strategies to generate equivalent ratios—e.g,* $\frac{1}{2}=\frac{1+1}{2+2}=\frac{2}{4}=\frac{2+1}{4+2}=\frac{3}{6}$ |
| Level 2 | Begins to Quantify | *Student's notion of ratio starts to develop, he can work with one ratio but may have trouble comparing to another ratio. May have additive misconception—e.g.,* $\frac{1}{2}=\frac{1+1}{2+1}=\frac{2}{3}$ |
| Level 1 | Has Intuitive Understanding | *Student has an intuitive understanding that allows qualitative comparisons of more or less—e.g., which drink is sweeter.* |

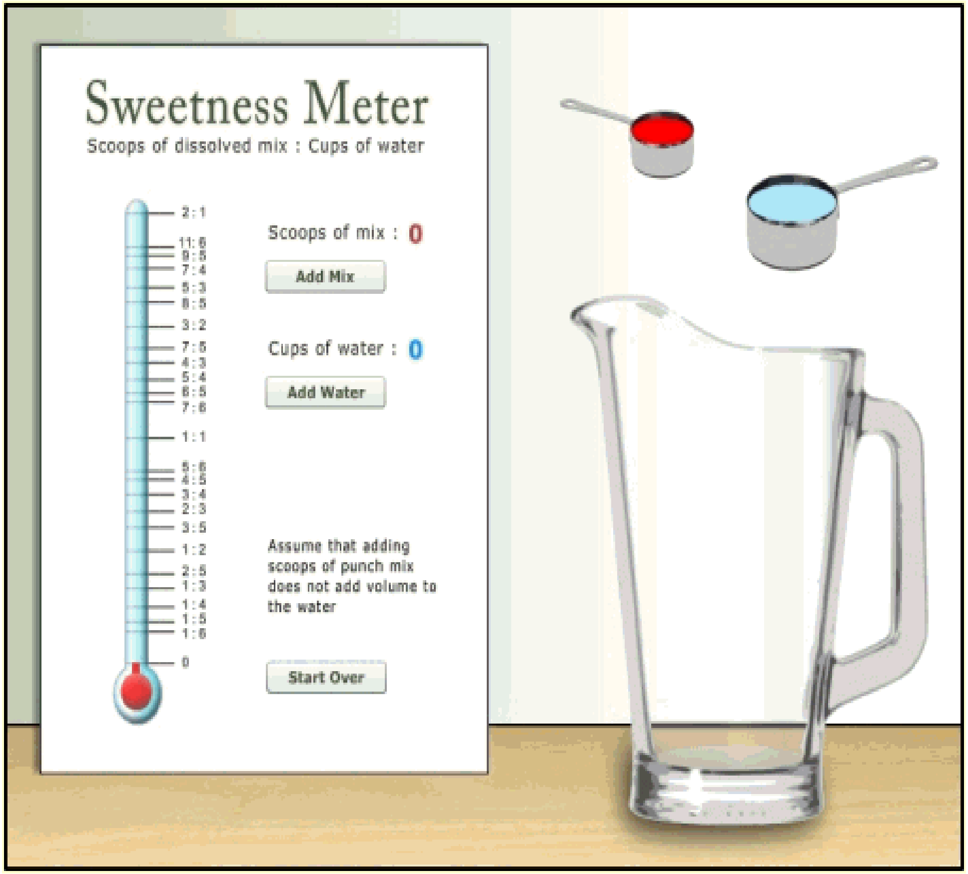
9

Source: Baxter and Junker, 2001 (cited in Weaver and Junker, 2004)

# Proportional Punch

Cherry punch can be made by adding sweet punch mix to water. In this task, you will see how ratios can be used to compare the sweetness of punch made using different recipes.

CBAL Example, ETS




A tool that simulates making punch and determining its sweetness is available for your use throughout the task. Try it out now by clicking on the icon. No more than 2 scoops of mix can dissolve in each cup of water, so if you add extra mix it will settle on the bottom of the pitcher and the sweetness meter will stay at 2:1.

CBAL Example, ETS

One cup of water is added to the punch in the pitcher, and no mix is added. The punch will be

- sweeter than it was.
- not as sweet as it was.
- just as sweet as it was.

Explain your answer.

CBAL Example, ETS

**Level 1: Has Intuitive Understanding**

Student's intuitive understanding of quantity permits them to answer questions about more and less (e.g., which drink is sweeter?) or fairness (e.g., divide pizza or cookies so everyone gets a fair share).

•Refer to the table below when answering the following questions.

•Use the sweetness meter to check your answers.

| | Number of | |
|---|---|---|
| | Scoops of Mix | Cups of Water |
| Recipe A | 1 | 3 |
| Recipe B | 2 | 3 |
| Recipe C | 3 | 4 |

**Level 2: Begins to Quantify**
Student's notion of ratio starts to develop, can work with one ratio but may have trouble comparing to another ratio. May have additive misconception.

4. Which recipe makes the sweeter punch?

Explain your answer in terms of **number** of scoops of mix and cups of water.

5. Which recipe makes the sweeter punch?

Recipe B

Recipe C

Explain your answer in terms of **number** of scoops of mix and cups of water.

CBAL Example, ETS

| Malcolm's Recipe | | |
|---|---|---|
| Number of | | Ratio of |
| Scoops of Mix | Cups of Water | Scoops of Mix<br>Cups of Water |
| 3 | 5 | 3<br>5 |
| 6 | 10 | 6<br>10 |
| 15 | 25 | 15<br>25 |
| 30 | 50 | 30<br>50 |

Malcolm's recipe for making punch uses a ratio of 3 scoops of punch mix to every 5 cups of water.

How many scoops of punch mix are needed to fill a 150 cup jug using Malcolm's punch recipe?


Show your work.

CBAL Example, ETS

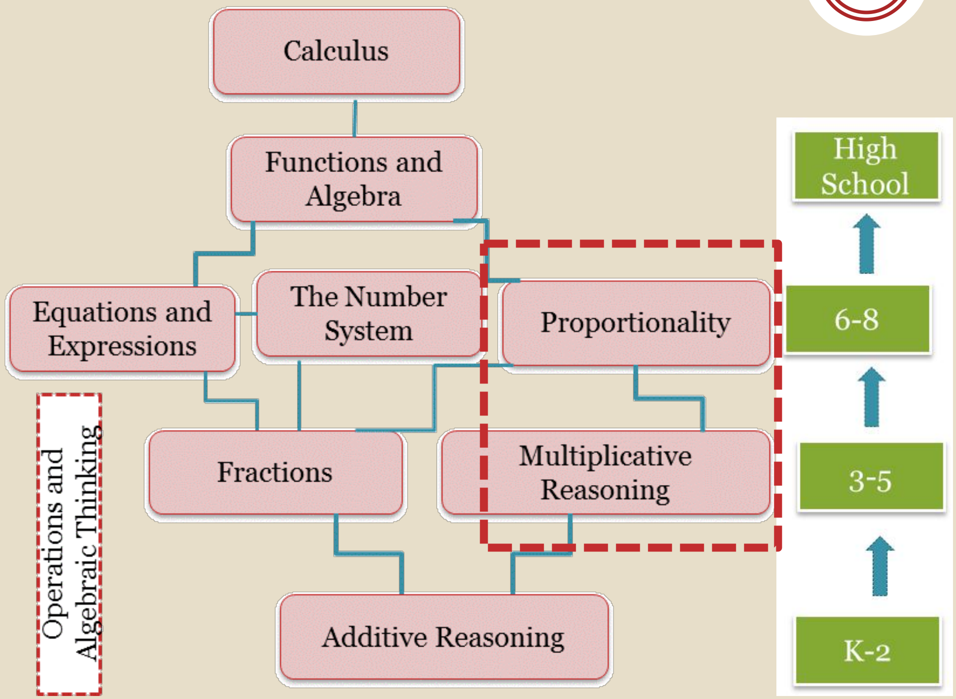
**90** scoops

Same answer doesn't mean same level of understanding:

**Level 3: Recognizes Multiplicative Relationships**
**OR**
**Level 4: Accommodates Covariance and Invariance**
**OR**
**Level 5: Has Generalized Model**

# The Opportunity:
# CBAL – OGAP - ILN Partnership






CBAL Proportional Reasoning

OGAP Multiplicative Reasoning

Year 1:  2 ILN sites nationally

Year 2:  More ILN sites nationally, and expand to fractions  at year 1 sites

# References




- Daro, Mosher, & Corcoran. (2011). Going from research to practice: Learning trajectories in action. *Mathematics Learning Trajectory Report*. Consortium for Policy Research in Education: Teacher's College, Columbia University.
- Clements, D., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, *6*(2), 81-89.
- Clements, D., & Sarama, J. (2009). *Learning and Teaching Early Math: The learning Trajectories Approach*. New York: Routledge.
- Petit in Daro, Mosher, & Corcoran. (2011). Going from research to practice: Learning trajectories in action. *Mathematics Learning Trajectory Report*. Consortium for Policy Research in Education: Teacher's College, Columbia University.
- Petit, Laird, and Marsden (2010). *A Focus on Fractions: Brining Research to the Classroom*, New York: Routledge
- VMP OGAP studies, 2004, 2005, 2007. The Vermont Mathematics Partnership
- CBAL, ETS Presentation to ILN Leaders, June 2012

# Related OGAP Publications




- Petit, Laird, and Marsden (2010), A Focus on Fractions: Brining Research to the Classroom. Routledge, New York and London.
- Petit, Laird, & Marsden (September, 2010). They get fractions as pies – but now what?. Mathematics in the Middle School, NCTM, Reston, Virginia.
- Petit, Zawojewski (2010). Formative Assessment in Elementary Classrooms. Teaching and Learning Mathematics: Translating Research for Elementary School Teachers. NCTM, Reston, VA.
- Petit, Zawojewski, Labaddo (2010). Formative Assessment in the Secondary School Classroom. Teaching and Learning Mathematics: Translating Research for Secondary School Teachers. NCTM, Reston, VA.
- Petit in Daro, Mosher, & Corcoran. (2011). Going from research to practice: Learning trajectories in action. *Mathematics Learning Trajectory Report*. Consortium for Policy Research in Education: Teacher's College, Columbia University (pp. 35-39). http://www.cpre.org/sites/default/files/researchreport/1220_learningtrajectoriesinmathcciireport.pdf
- Teachers College (2009). Charting Path to Learning, 2009 Annual Report. Teacher's College, Columbia University (pp. 30-35). http://www.tc.edu/news/pubs/annual2009/
- Ercole, Frantz, and Ashline (April 2011). Multiple Ways to Solve Proportional Reasoning Problems. Mathematics Teaching in the Middle School, 16:8, 482-4.

# For more information go to margepetit.com or contact...



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# For OGAP References go to….



- http://margepetit.com/

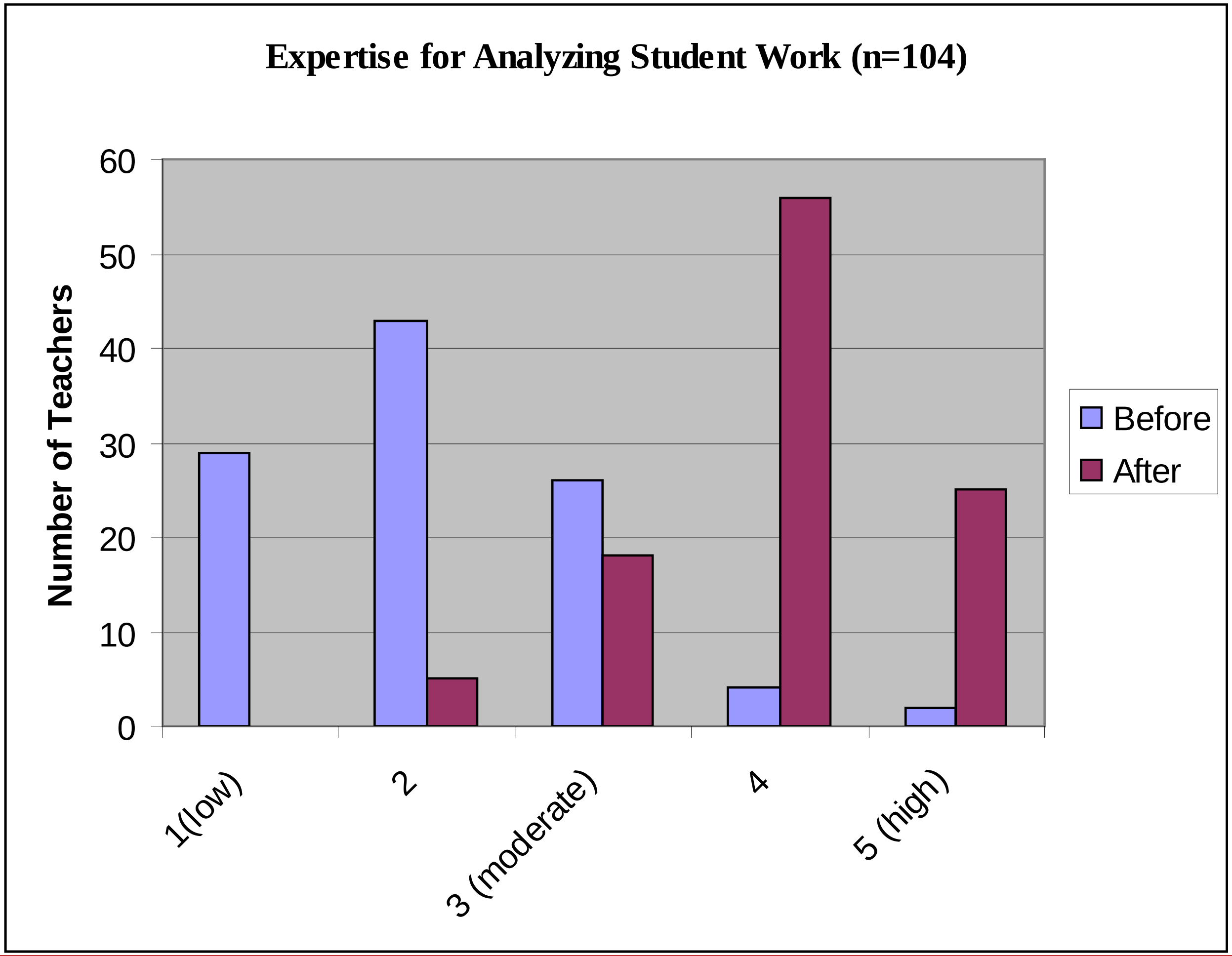
# Additional OGAP Data

What do *teacher leaders and teachers* say about their experience in relationship to the stated goals and the use of OGAP formative assessment system?

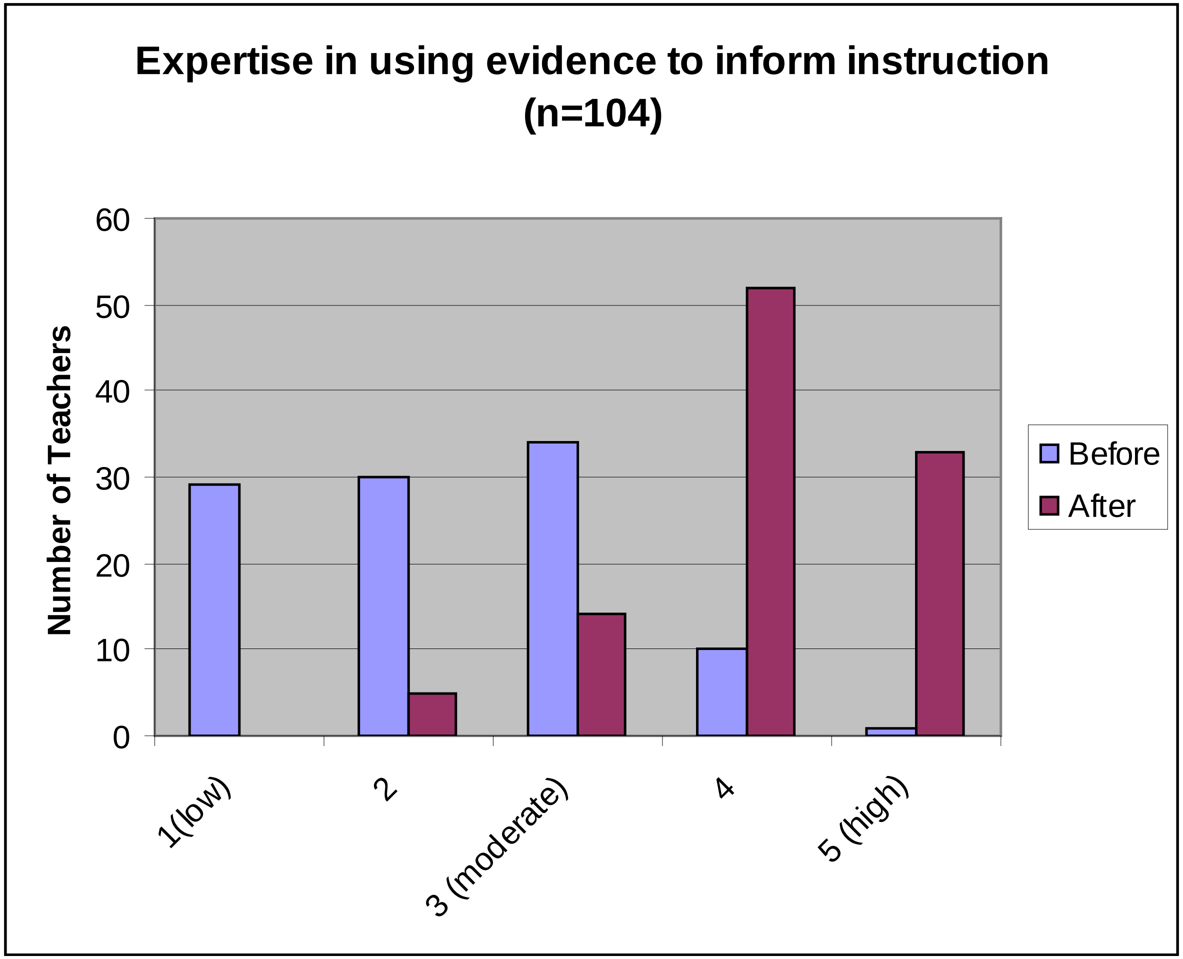
**Results based on a spring 2007 online survey**

# Expertise for analyzing student work (for evidence of developing understanding, common errors and misconceptions)…

**Before and After Experience**

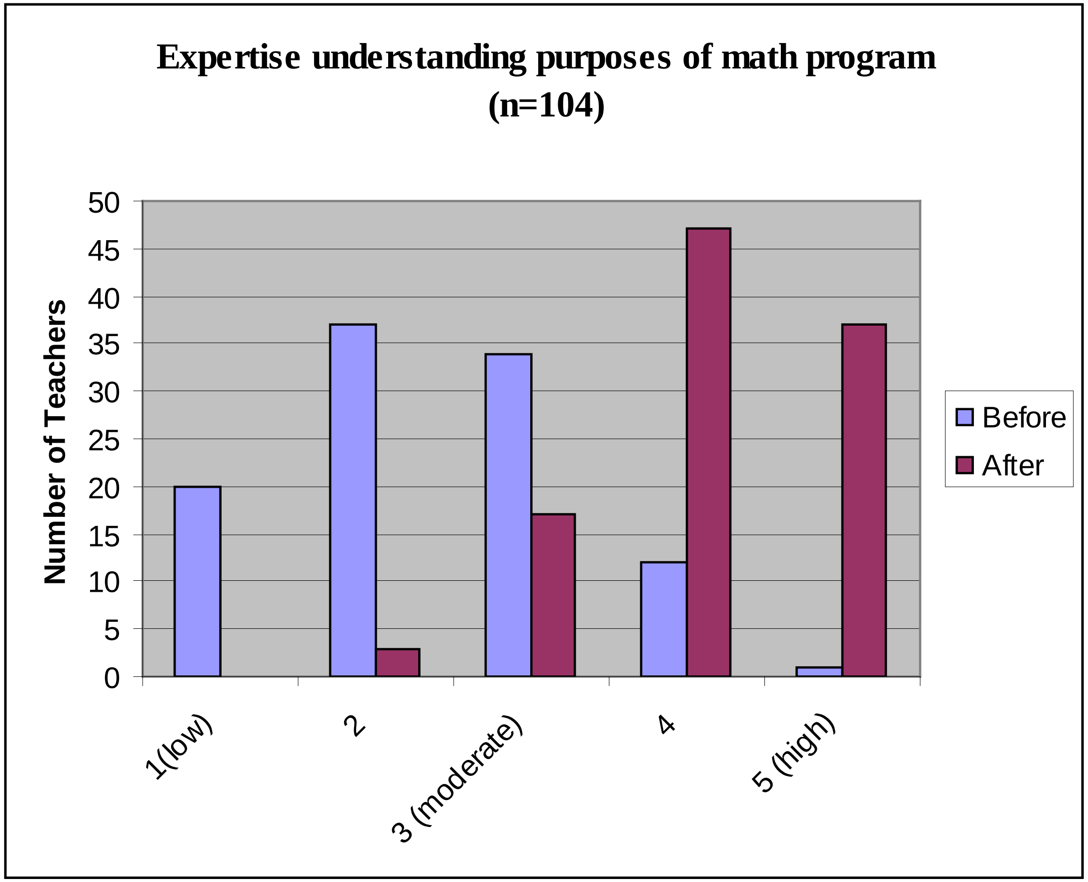


# Expertise in using evidence in student work to inform instruction…



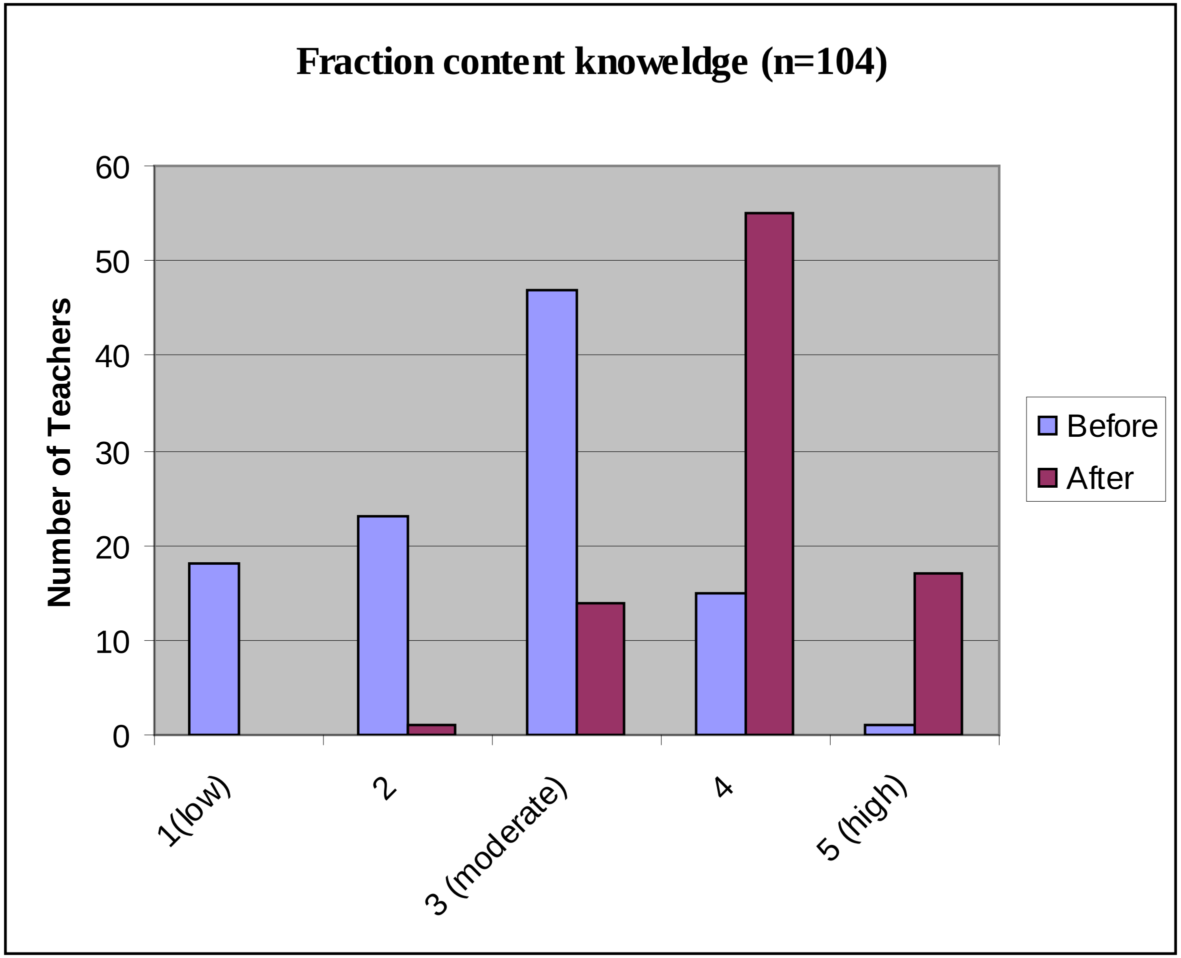
# Understanding purposes of activities in mathematics program…

## Before and After Experience



# Fraction content knowledge…

## Before and After Experience



# Pre-post Question – Pilot OGAP Teacher Assessment (2007)



Provide three strategies students can use to solve this problem. Provide examples.

1) Which fraction is closest to 1? Show your work.

$\frac{1}{2}$ $\frac{7}{9}$ $\frac{11}{13}$ $\frac{1}{6}$

Pilot OGAP Teacher Assessment Question

# Sample Teacher Responses



**Pre-assessment Q1 A**

① $\frac{1}{2} = \frac{117}{234}$  $\frac{7}{9} = \frac{182}{234}$  $\frac{11}{13} = \frac{198}{234}$

$\frac{1}{6} = \frac{39}{234}$  ∴ $\frac{11}{13}$ is closest to 1

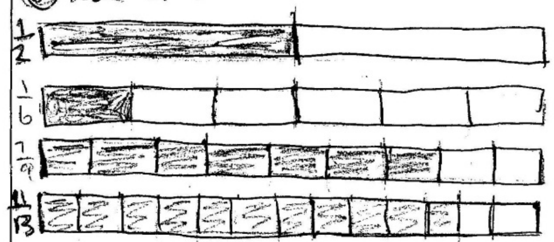
② Use fraction bars kit provided, (ninths + thirteenths are in it.)

③

**Post-assessment Q1 A**

① Unit fractions: $\frac{1}{2}, \frac{1}{6}$
sixths are smaller parts than halves.

② Use of area models




③ Use $\frac{1}{2}$ benchmark.
Using unit fraction reasoning, $\frac{1}{6}$ is smaller than $\frac{1}{2}$.
$\frac{7}{9}$ and $\frac{11}{13}$ are greater than $\frac{1}{2}$.

(continue on back as needed)

$\frac{11}{13}$ is $\frac{2}{13}$ away from 1 whole.
$\frac{7}{9}$ is $\frac{2}{9}$ away from the 1 whole.
Since 13ths are smaller, $\frac{11}{13}$ is closer to 1.

# Findings (Petit-Cunningham, 2008)



- Teacher leaders increased the range of strategies that they used pre to post to solve the two problems.

- Mentees also increased the range, but to a lesser degree

| Mentors and Mentees Pre - Post Teacher Assessment | | | |
|---|---|---|---|
| | Pre mean | Post mean | T-test (p-) Significance ($p < 0.05$) |
| Mentors (n=25) | 6.16 | 9.8 | 3.52E-08 |
| Mentees (n= 42) | 5.6 | 7.9 | 7.73E-06 |

# OGAP Development Team and National Advisory Board



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- Jean Ward, Bennington Rutland Supervisory Union
- Rebecca Young, Hardwick Schools

Plus about 250 Vermont and Alabama teachers and teachers and about 5000 students who participated in OGAP Exploratory Studies and 2006-2008 scale-up

## OGAP National Advisory Board

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- **Ed Silver**, University of Michigan
- **Judith Zawojewski**, Illinois Institute of Technology