Math 171 Proficiency Packet on
Whole Numbers

Section 1: Addition with Whole Numbers

The **counting** (or **natural**) **numbers** are defined as the set of numbers {1, 2, 3, 4,…}. We will extend the counting numbers to include 0 and get the set of **whole numbers,** which are defined as the set {0, 1, 2, 3, …}.

Terms of Addition

Addend
\+ Addend
Sum

Although the term addend will not be seen very much, sum will be seen later on.

Properties of Addition

The set of whole numbers is **closed** under addition. This means that if you take any two whole numbers and add them you will get a *unique* whole number.

Example 1: The set of odd whole numbers, {1, 3, 5, 7, …} is **not closed** under addition. In order for the set {1, 3, 5,…} to be closed under addition, you would have to be able to take any two numbers from the set, add them together and get another number in the set. In this example, if we add 3 and 1 we get 4 and 4 is not in {1, 3, 5, 7, …} .

If we used a number line to add all combinations of the numbers 0 through 9, we would get the following table. The table is read in the following manner: Suppose you wanted to add 5 + 7. First locate the 5 in the column on the left and the 7 in the row at the top. You would then read across from the 5 and down from the 7. The entry in the table that is across from 5 and below 7 is 12.

+	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	16	18

You can notice some facts about addition that are true regardless of the numbers involved. The first of these facts involves the number 0. Notice that whenever 0 is added to a number, the result is the original number. Since this fact is true no matter what number we add to 0, we call it the **Addition Property of Zero**.

ADDITION PROPERTY OF ZERO
If we let x represent any number, then it is always true that
$x + 0 = x$ and $0 + x = x$

A second property you can notice from the addition table, is that **the order of two numbers in a sum can be changed without changing the result**. This is called the **Commutative Property** of Addition and it is written in symbols as follows.

COMMUTATIVE PROPERTY OF ADDITON

If a and b are any two numbers then $a + b = b + a$

The commutative property states that the order you add two numbers is immaterial. For example, 8 + 3 represents the same number as 3 + 8.

****

Now You Try (Section 1.1)

Use the **Commutative Property** of addition to rewrite each sum.

a) $6 + 3 =$ _______________

b) $7 + 12 =$ _______________

c) $n + 9 =$ _______________

(Answers to **Now You Try** (Section 1.1) *are found on page 17.)*

****

Our operations are called **binary operations.** This means that no matter how many numbers you add or multiply, only two numbers are combined at any one time. Consider 2 + 3 + 4. You can add 2 and 3 and then add the result to 4. We can write this using parentheses as $2 + 3 + 4 = (2 + 3) + 4$. You could also have added the 3 to the 4 and then added the result to 2, which could be written as $2 + 3 + 4 = 2 + (3 + 4)$.

A third property that was illustrated above is the **Associative Property of Addition.** This property has to do with sums of three numbers. In words, it says that **changing the grouping of three numbers in a sum does not change the result**. Written in symbols, we have

ASSOCIATIVE PROPERTY OF ADDITION

If a, b and c are any three numbers then $(a + b) + c = a + (b + c)$

****

Now You Try (Section 1.2)

Use the **Associative Property** of addition to rewrite each sum.

a) $(3 + 8) + 5 =$ _______________

b) $7 + (5 + 8) =$ _______________

c) $6 + (2 + x) =$ _______________

(Answers to **Now You Try** (Section 1.2) *are found on page 17.)*

Algorithms for Addition with Whole Numbers

An algorithm is an organized step-by-step process that is used to reach a particular goal. There are many algorithms for whole number addition that you should be familiar with.

1. Expanded Notation Algorithm

Example 1: Add: 336 + 389

Solution: This type of addition is best done vertically. Using expanded notation, we write each number showing the place value of the digits.

$$\begin{array}{rcl} 336 & = & 300 + 30 + 6 \\ \underline{+\ 389} & = & \underline{300 + 80 + 9} \\ & & 600 + 110 + 15 \end{array}$$

$600 + 100 + 10 + 10 + 5$ (Rewriting 110 as 100 + 10 and 15 as 10 + 5)

$700 + 20 + 5$ (Adding the values in the hundreds and the tens places)

725 (Finally, adding all place values)

Example 2: Add: 184 + 364 + 273

Solution:

$$\begin{array}{rcl} 184 & = & 100 + 80 + 4 \\ 364 & = & 300 + 60 + 4 \\ \underline{+\ 273} & = & \underline{200 + 70 + 3} \\ & & 600 + 210 + 11 \end{array}$$

Since 210 = 200 + 10 and 11 + 10 + 1, we can write,

$600 + 200 + 10 + 10 + 1 = 600 + 200 + 20 + 1 = 800 + 20 + 1 = 821$

We can summarize this process by using shorthand notation for carrying:

$$\begin{array}{r} {}^{21} \\ 184 \\ 364 \\ \underline{+\ \ 273} \\ 821 \end{array}$$

Step 3 Sum of the digits in the hundreds column plus 2 carried from tens column is 8. → 821

821 ← **Step 1** Sum of the digits in the ones column is 11. Write 1 and carry the 1.

↑ **Step 2** Sum of the digits in the tens column plus 1 carried from the ones column is 22, write 2 and carry the 2.

This shorthand notation is our **traditional addition algorithm**. It is easier than writing out the numbers showing the place value of the digits.

2. Scratch Addition Algorithm

Begin by adding from the top down in the units column. When you add a digit that makes your sum 10 or more, scratch out the digit and note the units digit of your present sum. Start with the digit noted and continue adding and scratching until you have completed the units column, writing down the units digit of the last sum as the units digit of the answer. Now, count the number of scratches in the units column and, starting with this number, add on down the tens column, repeating the scratch process as you go. Continue the entire process until all the columns have been added. This gives the desired answer.

Example 1:

$$\begin{array}{r} {}^{1}2\;1^{2}\;0\;9 \\ 3\;0\;\cancel{8}_{7} \\ 4\;\cancel{6}\;1\;\cancel{5}_{2} \\ +\quad 4\;2 \\ \hline 7\;0\;7\;4 \end{array}$$

Example 2:

$$\begin{array}{r} {}^{1}4\;{}^{1}5\;{}^{1}6\;\;0 \\ \cancel{5}_{4}\;1\;\;2 \\ 1\;0\;\;\cancel{7}_{5}\,\cancel{9}_{1} \\ +\quad 2\;\;5 \\ \hline 6\;4\;7\;6 \end{array}$$

3. Partial Sums Algorithm

2 4 7		
+ 5 8		
1 5	(7 + 8 = 15)	add ones column
9 0	(50 + 40 = 90)	add tens column
2 0 0	(200 + 0 = 200)	add hundreds column
3 0 5		use the traditional algorithm to add the partial sums, finding the total sum

****

Now You Try (Section 1.3)

Add the following numbers, using the three algorithms discussed above. 3 5 7 9 + 7 1 3 + 4 0 0 5 + 9 4

1. Expanded notation algorithm

2. Scratch Addition Algorithm

3. Partial Sums Algorithm

(Answers to **Now You Try** (Section 1.3) *are found on page 17.)*

Section 2: Subtraction with Whole Numbers

Terms of Subtraction

$$\begin{array}{rl} & \text{Minuend} \\ - & \underline{\text{Subtrahend}} \\ & \text{Difference} \end{array}$$

Once again, only the answer difference, will be seen later on.

Properties of Subtraction

Definition: Whole Number Subtraction

For any whole numbers a and b with b < a, a - b = c if and only if c + b = a for a unique whole number c.

When we want to subtract 4 from 9, we write 9 - 4. According to the definition above,

9 - 4 = ? is the same as ? + 4 = 9.

In both cases we are looking for the number we add to 4 to get 9. The number we are looking for is 5. We have two ways to write the same statement.

Subtraction		**Addition**
9 - 4 = 5	or	5 + 4 = 9

For every subtraction problem, there is an equivalent addition problem.

Example 1:

	Subtraction		**Addition**
a)	8 - 2 = 6	because	6 + 2 = 8
b)	14 - 8 = 6	because	6 + 8 = 14
c)	16 - 11 = 5	because	5 + 11 = 6

To subtract numbers with two or more digits, we align the numbers vertically and subtract in columns.

Algorithms for Subtraction with Whole Numbers

Example 1: Subtract: 582 - 371

Solution: We proceed as we did with addition by using expanded notation.

$$\begin{array}{rcl} 582 & = & 5 \text{ hundreds} + 8 \text{ tens} + 2 \text{ ones} \\ \underline{-\ 371} & = & \underline{3 \text{ hundreds} + 7 \text{ tens} + 1 \text{ one}} \\ & & 2 \text{ hundreds} + 1 \text{ tens} + 1 \text{ one} \end{array}$$

←*Subtract the bottom number in each column from the number above it.*

The difference is 2 hundreds + 1 ten + 1 one, which we write in standard form as 211.

When the bottom digit in any column is larger than the digit above it, subtraction will involve **borrowing**. Borrowing can be thought of as the reverse of the carrying we did in addition.

Example 2: Subtract: 83 - 27

Solution: As we did in the last example, we write the problem vertically and in expanded notation.

$$\begin{array}{rcl} 83 & = & 8 \text{ tens } + \ 3 \text{ ones} \\ \underline{-\ \ 27} & = & \underline{2 \text{ tens } + \ 7 \text{ ones}} \end{array}$$

In the ones column, we cannot subtract immediately because the 7 is larger than 3. Instead, we borrow 1 ten from the 8 tens in the tens column. We can rewrite the number 83 as

$$\begin{array}{rcl} 8 \text{ tens} + 3 \text{ ones} & = & 7 \text{ tens } + \ 1 \text{ ten } + \ 3 \text{ ones} \\ & = & 7 \text{ tens } + \ 10 \text{ ones } + \ 3 \text{ ones} \\ & = & 7 \text{ tens } + \ 13 \text{ ones} \end{array}$$

Now we can subtract.

$$\begin{array}{rcccl} 83 & = & 8 \text{ tens } + \ 3 \text{ ones} & = & 7 \text{ tens } + \ 13 \text{ ones} \\ \underline{-\ \ 27} & = & \underline{2 \text{ tens } + \ 7 \text{ ones}} & = & \underline{2 \text{ tens } + \ \ 7 \text{ ones}} \\ & & & & 5 \text{ tens } + \ \ 6 \text{ ones} \end{array}$$

The result is 5 tens + 6 ones, which can be written in standard form as 56.
The shorthand form for this problem looks like:

$$\begin{array}{r} {}^{7}\cancel{8}\ \ \cancel{3}^{13} \\ -\ 2\ \ \ 7 \\ \hline 5\ \ \ 6 \end{array}$$

(The 13 shows we borrowed 1 tens to add to the 3 ones.)

Remember to check your work. 83 - 27 = 56 because 56 + 27 = 83.

* * *

Now You Try (Section 2)

1. Subtract 37 from 124 using both expanded notation and the shorthand form.

2. Use examples to decide whether the following properties *(which are true for addition of whole numbers)* are also true for subtraction of whole numbers.

   a) commutative property

   b) associative property

(Answers to **Now You Try** (Section 2) *are found on page 17 & 18.)*

Section 3: Multiplication with Whole Numbers

The following table gives the multiplication facts up through 9 x 9 = 81. This table is read the same way we read the addition table earlier.

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Terms of Multiplication

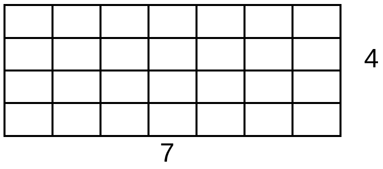
Factor
x Factor
Product

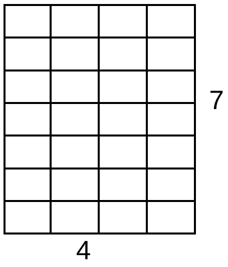
In this case, both factor and product will be seen later on.

Properties of Whole Number Multiplication

Before looking at the multiplication algorithm, let's look at the properties of whole number multiplication. When you multiply any two whole numbers together you **always** get another whole number. Therefore the set of whole numbers is **closed** under multiplication.

The **commutative property of multiplication** says that if a and b are any whole number, then a · b = b · a. This is easy to demonstrate using a rectangular array. The following shows that 4 · 7 = 7 · 4 because each array has 28 boxes.





The number 1 is the unique whole number for which b · 1 = 1 · b = b holds for all whole numbers b. The number 1 is called the **multiplicative identity**. There is also the **multiplication property of 0** that for all whole numbers b, 0 · b = b · 0 = 0.

Multiplication is **associative**, which means that if a, b, and c are any whole numbers, then a · (b · c) = (a · b) · c. The most important property is the **distributive property**, which relates multiplication and addition. It says that if a, b, and c are any three whole numbers, then a · (b + c) = a · b + a · c.

The properties of multiplication are summarized below.

Multiplication Property of 0

If a represents any number, then $a \cdot 0 = 0$ and $0 \cdot a = 0$

Multiplication by 0 always results in 0.

Multiplication Property of 1

If a represents any number, then $a \cdot 1 = a$ and $1 \cdot a = a$.

Multiplying any number by 1 leaves the number unchanged.

Commutative Property of Multiplication

If a and b are any two numbers, then $ab = ba$.

We can change the order of the numbers in a product without changing the result.

Associative Property of Multiplication

If a, b, and c represent any three numbers, then $(ab)c = a(bc)$.

We can change the grouping of the numbers in a product without changing the result.

Distributive Property

If a, b, and c represent any three whole numbers, then $a(b + c) = ab + ac$.

****

Now You Try (Section 3.1)

State the property that each example illustrates.

1) $8 \cdot (5 \cdot 3) = (8 \cdot 5) \cdot 3$

2) $7 \cdot 0 = 0$

3) $9 \cdot 8 = 8 \cdot 9$

4) $7(4 + 6) = 7 \cdot 4 + 7 \cdot 6$

5) $1 \cdot 19 = 19$

(Answers to **Now You Try** (Section 3.1) *are found on page 18.)*

Algorithms for Multiplication of Whole Numbers

Partial Products. This algorithm is based on expanded notation and the distributive property of multiplication over addition. The following example illustrates this method.

Example 1: Multiply: 750 by 12.

Solution: 750 x 12 = 750 x (2 + 10) = (750 x 2) + (750 x 10)

= 1500 + 7500

= 9000

We can write this version of partial sums alternately as follows:

$$\begin{array}{rl} 750 & \qquad \textit{(traditional shortcut method)} \\ \underline{\text{x} \quad 12} & \\ 1500 & \leftarrow (2 \text{ x } 750) \\ \underline{+\ 7500} & \leftarrow (10 \text{ x } 750) \\ 9000 & \end{array}$$

Note: When students are taught this algorithm they usually do not write the final zero in 7500.

Example 2: Multiply: 163 by 37.

Solution: Again we will show both methods. Study these solutions until you understand the relationship between the two methods.

163 x 37 = 163 x (7 + 30) = (163 x 7) + (163 x 30)

= 1141 + 4890

= 6031

Shortcut method:

$$\begin{array}{rl} 163 & \\ \underline{\text{x} \ \ 37} & \\ 1141 & \leftarrow (7 \text{ x } 163) \\ \underline{4890} & \leftarrow (30 \text{ x } 163) \\ 6031 & \end{array}$$

Example 3: Multiply: 73 by 46.

Solution: Again we will show both methods. Study these solutions until you understand the relationship between the two methods.

This is the same as 73(40 + 6) or 73(40) + 73(6).

We can find each of these products by using the shortcut method:

$$\begin{array}{rcr} 1 & \qquad\qquad & 1 \\ 7\,3 & & 7\,3 \\ \underline{\times\ 4\,0} & & \underline{\times\ \ 6} \\ 2\,9\,2\,0 & & 4\,3\,8 \end{array}$$

The sum of these two numbers is 2920 + 438 = 3358. Here is a summary of what we have so far:

$$\begin{aligned} 73(46) &= 73(40+6) && \textit{46 = 40 + 6} \\ &= 73(40) + 73(6) && \textit{distributive property} \\ &= 2920 + 438 && \textit{multiply} \\ &= 3358 && \textit{add} \end{aligned}$$

The shortcut form for this problem is

$$\begin{array}{rcl} 7\ 3 & & \\ \underline{\times\ 4\ 6} & & \\ 4\ 3\ 8 & \leftarrow & 6(73) = 438 \\ \underline{2\ 9\ 2\ 0} & \leftarrow & 40(73) = 2920 \\ 3\ 3\ 5\ 8 & & \end{array}$$

You should go over the preceding examples in detail so you understand the reasons behind the process of multiplication. Besides being able to do multiplication, you should understand it.

****

Now You Try (Section 3.2)

Do the following multiplication problems using partial products and the traditional shortcut method.

1) 8(57)

2) 4(769)

3) 56(83)

(Answers to **Now You Try** (Section 3.2) *are found on page 18)*

Section 4: Division with Whole Numbers

Terms of Division

$$\begin{array}{r} \text{quotient} \\ divisor\overline{)\,dividend} \\ -\ \underline{\qquad\qquad} \\ \text{remainder} \end{array}$$

Here all terms will be used, but quotient will be seen the most.

The traditional algorithm for division is illustrated in the next two examples.

Example 1: Divide: 476 ÷ 7

Solution: Since 7(6) = 42, our first estimate of the number of sevens that can be subtracted from 47 is 6:

$$\begin{array}{rll} 6 & & \\ 7\overline{)476} & & \\ \underline{42} & \leftarrow & \textit{7(6) = 42, place the 6 in the quotient above the tens column} \\ 56 & \leftarrow & \textit{47 - 42 = 5; then bring down the 6} \end{array}$$

Since 7(8) = 56, we have

$$\begin{array}{rll} 68 & & \\ 7\overline{)476} & & \\ \underline{42} & & \\ 56 & & \\ \underline{56} & \leftarrow & \textit{7(8) = 56, place the8 in the quotient above the ones column} \\ 0 & \leftarrow & \textit{56 - 56 = 0} \end{array}$$

Our result is 476 ÷ 7 = 68, which we can check with the following equation:

$$\boxed{\text{(divisor x quotient) + remainder = dividend}}$$

$$\begin{array}{r} {}^{5} \\ 68 \\ \underline{\times\ \ 7} \\ 476 \\ \underline{+\ \ 0} \\ 476 \end{array}\quad \surd$$

Example 2: Divide: 33,997 ÷ 56

Solution: Since 50(6) = 300, our first estimate of the number of fifty-sixes that can be subtracted from 339 is 6.

$$\begin{array}{r} 6 \\ 56\overline{)33997} \end{array}$$

$$\begin{array}{r} -\underline{336} \\ 39 \end{array}$$

← *56(6) = 336, place the6 in the quotient above the hundreds column*

← *339 - 336 = 3, then bring down the 9*

Since 59 cannot be divided into 39, we write a 0 in the quotient in the tens column.

$$\begin{array}{r} 607 \\ 56\overline{)33997} \end{array}$$

$$\begin{array}{r} -\underline{3\ 3\ 6} \\ 3\ 9\ 7 \\ \underline{3\ 9\ 2} \\ 5 \end{array}$$

← *bring down the 7*

← *56(7) = 392, place the7 in the quotient above the ones column*

← *397 - 392 = 5, write 5 as the remainder.*

Therefore, 33,997 ÷ 56 = 607 R 5.

Check: 56(607) + 5 = 33992 + 5 = 33997 √

****

<u>Now You Try</u> (Section 4)

1. Divide 274 by 8.

2. Divide 1238 by 23.

3. Use examples to decide whether or not the following properties (which are true for multiplication of whole numbers) are true for division of whole numbers.

   a) division property of 0

   b) division property of 1

   c) commutative property of division

   d) associative property of division

(Answers to **Now You Try** (Section 4) *are found on page 19.)*

****

Section 5: Finding the average

To find the average of a set of numbers, add all the numbers together, then divide by the amount of numbers in the set.

Example 1: Find the average of 82, 75, 60, 43, and 90

Solution: Add: 82 + 75 + 60 + 43 + 90 = 350

Divide: 350 ÷ 5 = 70 .

Now You Try (Section 5)

1. Find the average of 76, 49, 62, 55, 24, and 88.

(Answers to **Now You Try** (Section 5) *are found on page 19)*

****

Exercises for Whole Numbers

Do all problems on a separate piece of paper, showing all work.

1. Add the following using expanded notation:

   a) 842 + 306

   b) 391 + 56 + 275

2. Use the traditional algorithm to add the following:

   a) 649 + 743

   b) 46,862 + 472 + 948

   c) Find the sum of 573, 4,009, 38, 4,612

3. Add the following using the scratch algorithm:

$$\begin{array}{r} 4\ 7\ 2\ 4 \\ 6\ 7\ 3 \\ 5\ 7\ 0\ 7 \\ 9\ 6\ 0\ 0 \\ \underline{+ \qquad 9\ 5} \end{array}$$

4. Use expanded notation to find the difference between 19,045 and 7,937. Show that the answer checks. For example, 24 - 7 = 17 because 17 + 7 = 24.

5. Do the following subtractions:

   a) $\begin{array}{r} 7,\ 9\ 0\ 5 \\ \underline{-\ 1,\ 9\ 8\ 2} \end{array}$

   b) $\begin{array}{r} 1\ 3\ 6,\ 0\ 0\ 0 \\ \underline{-\quad 6\ 7,\ 1\ 4\ 2} \end{array}$

6. Do the following multiplications using partial products and the shortcut (traditional) method.

   a) 43(78)

   b) 60(745)

7. Divide 18,395 by 27. Show that the answer checks.

8. State the property that each example illustrates.

a) $5 + 8 = 8 + 5$

b) $7 \cdot 1 = 7$

c) $6 \cdot (2 \cdot 4) = (6 \cdot 2) \cdot 4$

d) $0 \cdot 8 = 0$

e) $9(3 + 2) = 9 \cdot 3 + 9 \cdot 2$

f) $6 + 0 = 6$

9) Find the average of:

26, 24, 28, 29, 27, 25, 24, and 23.

Answers to <u>Now You Try</u>

Section 1.1:

a) 3 + 6 b) 12 + 7 c) 9 + n

Section 1.2:

a) 3 + (8 + 5) b) (7 + 5) + 8 c) (6 + 2) + x

Section 1.3:

1.

$$\begin{array}{rrrrrrr} 3000 & + & 500 & + & 70 & + & 9 \\ & & 700 & + & 10 & + & 3 \\ 4000 & & & & & + & 5 \\ \hline & & & & 90 & + & 4 \end{array}$$

$$7000 + 1200 + 170 + 21 = 7000 + 1000 + 200 + 100 + 70 + 20 + 1$$

$$= 8000 + 300 + 90 + 1 = 8391 .$$

2.

$$\begin{array}{r} {}^{1}3\ {}^{1}5^{2}7\ 9 \\ \cancel{7}\ {}_{3}\cancel{1}_{0}\ \cancel{3}_{2} \\ 4\ 0\ 0\ 5 \\ +\qquad 9\ \cancel{4} \\ \hline 8\ 3\ 9\ 1 \end{array}$$

3.

$$\begin{array}{rl} 3\,5\,7\,9 & \\ 7\ 1\ 3 & \\ 4\,0\,0\,5 & \\ +\quad 9\ 4 & \\ \hline 2\ 1 & (\textit{ones place}) \\ 1\ 7\ 0 & (\textit{tens place}) \\ 1\,2\,0\ 0 & (\textit{hundreds place}) \\ +\,7\,0\,0\,0 & (\textit{thousands place}) \\ \hline 8\ 3\ 9\,1 & \end{array}$$

Section 2:

1.

$$\begin{array}{ccccc} 100 + 20 + 4 & = & 100 + 10 + 14 & = & 0 + 110 + 14 \\ \underline{-\qquad 30 + 7} & & \underline{-\qquad 30 + 7} & = & \underline{-\qquad 30 + 7} \end{array}$$

$$80 + 7 = 87 . \qquad \text{or} \qquad \begin{array}{r} 1\ {}^{1}\cancel{2}\ {}^{1}4 \\ -\quad 3\ 7 \\ \hline 8\ 7 \end{array}$$

2. a) Subtraction is <u>not</u> commutative. For example,

$$3 - 5 \overset{?}{=} 5 - 3$$
$$-2 \neq 2$$

b) Subtraction is <u>not</u> associative. For example,

$$(3 - 2) - 1 \overset{?}{=} 3 - (2 - 1)$$
$$1 - 1 \overset{?}{=} 3 - 1$$
$$0 \neq 2$$

Section 3.1:

1. associative property of multiplication
2. multiplication property of 0
3. commutative property of multiplication
4. distributive property
5. multiplication property of 1

Section 3.2:

1.

$$\begin{aligned} 8(50 + 7) &= 8 \cdot 50 + 8 \cdot 7 \\ &= 400 + 56 \\ &= 456 \ . \end{aligned} \qquad \text{or} \qquad \begin{array}{r} {}^{5}57 \\ \underline{\times \quad 8} \\ 456 \ . \end{array}$$

2.

$$\begin{aligned} 4(700 + 60 + 9) &= 4 \cdot 700 + 4 \cdot 60 + 4 \cdot 9 \\ &= 2800 + 240 + 36 \\ &= 3076 \ . \end{aligned} \qquad \text{or} \qquad \begin{array}{r} {}^{2}7{}^{3}69 \\ \underline{\times \quad 4} \\ 3076 \ . \end{array}$$

3.

$$\begin{aligned} 56(80 + 3) &= 56 \cdot 80 + 56 \cdot 3 \\ &= 4480 + 168 \\ &= 4648 \ . \end{aligned} \qquad \text{or} \qquad \begin{array}{r} {}^{1}56 \\ \underline{\times\ 83} \\ 168 \\ \underline{4480} \\ 4648 \ . \end{array}$$

Section 4:

1.

$$\begin{array}{rl} & 34 \\ 8 & \overline{)274} \\ & \underline{24} \\ & 34 \\ & \underline{32} \\ & 2 \end{array}$$

so 247 ÷ 8 = 34 R 2 .

2.

$$\begin{array}{rl} & 53 \\ 23 & \overline{)1238} \\ & \underline{115} \\ & 88 \\ & \underline{69} \\ & 19 \end{array}$$

so 1238 ÷ 23 = 53 R 19 .

3. a) No, can't divide by zero.

$8 \div 0 =$ undefined

b) Yes, any number divided by 1 equals that number.

$8 \div 1 = 8.$

c) No, division is not commutative.

$$15 \div 3 \overset{?}{=} 3 \div 15$$

$$5 \neq \frac{1}{5}$$

d) No, division is not associative.

$$30 \div (10 \div 5) \overset{?}{=} (30 \div 10) \div 5$$

$$30 \div 2 \overset{?}{=} 3 \div 5$$

$$15 \neq \frac{3}{5}$$

Section 5:

1. $76 + 49 + 62 + 55 + 24 + 88 = 354$

$354 \div 6 = 59$.